# Solution to Practice 4g

**B1** The matrices with a determinant not equal to zero (and therefore invertible) are (a),(c),(d),and (e). The matrices for (b) and (f) have determinant zero, and therefore are not invertible.

**B3(a)** $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 2 \\ p & 1 & -2 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - pR_1 \end{matrix} \sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & 1 & -2+p \end{bmatrix} \begin{matrix} \\ \\ R_3 - R_2 \end{matrix}$

$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 4 \\ 0 & 0 & -6+p \end{bmatrix} = B.$

We have det$A$ = det$B = (1)(1)(-6+p) = -6+p$. And thus $A$ is invertible if and only if det$A \neq 0$, if and only if $-6+p \neq 0$m, if and only if $p \neq 6$.

**B3(b)** $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & p & p \\ 2 & 2 & 1 \end{bmatrix} \begin{matrix} \\ R_2 - 2R_1 \\ R_3 - 2R_1 \end{matrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & p-2 & p-2 \\ 0 & 0 & -1 \end{bmatrix} = B.$

We have det$A$ = det$B = (1)(p-2)(-1) = -p+2$. And thus $A$ is invertible if and only if det$A \neq 0$, if and only if $-p+2 \neq 0$, if and only if $p \neq 2$.

**B3(c)** $A = \begin{bmatrix} 2 & 5 & 2 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 4 & 2 \\ 1 & 0 & 1 & p \end{bmatrix} \begin{matrix} R_1 \updownarrow R_4 \\ \\ \\ \\ \end{matrix} \sim \begin{bmatrix} 1 & 0 & 1 & p \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 4 & 2 \\ 2 & 5 & 2 & 4 \end{bmatrix} \begin{matrix} \\ \\ \\ R_4 - 2R_1 \end{matrix}$

$\sim \begin{bmatrix} 1 & 0 & 1 & p \\ 0 & 1 & -1 & 1 \\ 0 & 1 & 4 & 2 \\ 0 & 5 & 0 & 4-2p \end{bmatrix} \begin{matrix} \\ \\ R_3 - R_2 \\ R_4 - 5R_2 \end{matrix} \sim \begin{bmatrix} 1 & 0 & 1 & p \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 5 & -1-2p \end{bmatrix} \begin{matrix} \\ \\ \\ R_4 - R_3 \end{matrix}$

$\sim \begin{bmatrix} 1 & 0 & 1 & p \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & -2-2p \end{bmatrix} = B.$

We have det$A = -$det$B = -(1)(1)(5)(-2-2p) = 10+10p$. And thus $A$ is invertible if and only if det$A \neq 0$, if and only if $10+10p \neq 0$, if and only if $p \neq -1$.

**B3(d)** $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \\ 1 & 4 & 16 & p \end{bmatrix} \begin{matrix} \\ R_2 - R_1 \\ R_3 - R_1 \\ R_4 - R_1 \end{matrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 \\ 0 & 2 & 8 & 26 \\ 0 & 3 & 15 & p-1 \end{bmatrix} \begin{matrix} \\ \\ R_3 - 2R_2 \\ R_4 - 3R_2 \end{matrix}$

$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 2 & 12 \\ 0 & 0 & 6 & p-22 \end{bmatrix} \begin{matrix} \\ \\ \\ R_4 - 3R_3 \end{matrix} \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 7 \\ 0 & 0 & 2 & 12 \\ 0 & 0 & 0 & p-58 \end{bmatrix} = B.$

We have $\det A = \det B = (1)(1)(2)(p-58) = 2p - 116$. And thus $A$ is invertible if and only if $\det A \neq 0$, if and only if $2p - 116 \neq 0$, if and only if $p \neq 58$.