Solution to Practice 1e

A7(c) $\vec{x} = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} + t \begin{bmatrix} 4 \\ -2 \\ -11 \end{bmatrix}, \ t \in \mathbb{R}$

A7(d) $\vec{x} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}, \ t \in \mathbb{R}$

A8(c) I will use the point P and direction vector $\vec{PQ} = \begin{bmatrix} -2-1 \\ 1-3 \\ 0-(-5) \end{bmatrix} = \begin{bmatrix} -3 \\ -2 \\ 5 \end{bmatrix}$. Then an equation for the line is $\vec{x} = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix} + t \begin{bmatrix} -3 \\ -2 \\ 5 \end{bmatrix}, \ t \in \mathbb{R}$.

A8(d) I will use the point P and direction vector $\vec{PQ} = \begin{bmatrix} 4-(-2) \\ 2-1 \\ 2-1 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix}$. Then an equation for the line is $\vec{x} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 6 \\ 1 \\ 1 \end{bmatrix}, \ t \in \mathbb{R}$.

A8(e) I will use the point P and direction vector $\vec{PQ} = \begin{bmatrix} -1-1/2 \\ 1-1/4 \\ 1/3-1 \end{bmatrix} = \begin{bmatrix} -3/2 \\ 3/4 \\ -2/3 \end{bmatrix}$. Then an equation for the line is $\vec{x} = \begin{bmatrix} 1/2 \\ 1/4 \\ 1 \end{bmatrix} + t \begin{bmatrix} -3/2 \\ 3/4 \\ -2/3 \end{bmatrix}, \ t \in \mathbb{R}$.

A10(a) Let P, Q and R be three points. Then another way to phrase this question is to ask whether or not the line through P and Q is the same as the line through P and R. The vector equations of these lines are $\vec{x} = \vec{p} + t\vec{PQ}$ ($t \in \mathbb{R}$) and $\vec{x} = \vec{p} + s\vec{PR}$ ($s \in \mathbb{R}$), respectively. Well, given that the "point" part of these equations are the same, we have that these equations describe the same line if and only if the direction vectors are non-zero scalar multiples of each other. As such, we have that P, Q and R are collinear if and only if $\vec{PQ} = s\vec{PR}$ for some $s \in \mathbb{R}$. (Note that this proof does not use specific properties of $\mathbb{R}^2$ or $\mathbb{R}^3$, and thus works in both settings.)

A10(b) We have $\vec{PQ} = \begin{bmatrix} 4-1 \\ 1-2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$, and $\vec{PR} = \begin{bmatrix} -5-1 \\ 4-2 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$. Since $\vec{PQ} = -\frac{1}{2}\vec{PR}$, we have that the points ARE collinear.

A10(c) We have $\vec{ST} = \begin{bmatrix} 3-1 \\ -2-0 \\ 3-1 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$ and $\vec{SU} = \begin{bmatrix} -3-1 \\ 4-0 \\ -1-1 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ -2 \end{bmatrix}$. As there is no $s \in \mathbb{R}$ such that $\vec{ST} = s\vec{SU}$, the three points are not collinear.