Solution to Practice 1b

A7(a) $\vec{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} + t \begin{bmatrix} -5 \\ 1 \end{bmatrix}$, $t \in \mathbb{R}$ (Sorry, but there really isn't any more explanation to be given...)

A7(b) $\vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + t \begin{bmatrix} -4 \\ -6 \end{bmatrix}$, $t \in \mathbb{R}$ (again, no explanation needed)

A9(a) Since this line is given in point-slope form, we know that it goes through the point (0,2) and has slope 3. Translating "slope 3" into the direction "up 3, right 1", we get a direction vector of $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$. (Remember to put the x_1 "right" direction first!). So, this is the equation of a line through $(0, 2)$ with direction vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$, so it has the vector equation $\vec{x} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 1 \\ 3 \end{bmatrix}$, $t \in \mathbb{R}$. Breaking this up into components, we get the parametric equations

$$\begin{aligned} x_1 &= 0 + t \text{ (or simply } x_1 = t) \\ x_2 &= 2 + 3t \end{aligned} \qquad t \in \mathbb{R}$$

A9(b) Those familiar with this form of the equation of a line will know simply by looking at it that it is a line with slope -3/2 with x_2-intercept $(0, 5/3)$. Looking a little closer will reveal that the point $(1, 1)$ satisfies the equation of the line, and if for no reason other than the fact that fractions are annoying, I choose to use that point instead. For similar reasons, instead of thinking of the slope as "up 1, left 2/3", I will instead translate it as "down 2, right 3", which corresponds to a direction vector of $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Again, remember to match up the "down" quantity with the x_2 component, and the "right" quantity with the x_1 component. (Not familiar with this format of the equation of a line? It isn't very difficult to solve for x_2, getting the equation into point-slope form. The resulting equation would be $x_2 = (-2/3)x_1 + 5/3$.)

So, we are first looking for the vector equation of a line through the point $(1, 1)$ with direction vector $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$, which is $\vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $t \in \mathbb{R}$. Breaking this up into components, we get the parametric equations

$$\begin{aligned} x_1 &= 1 + 3t \\ x_2 &= 1 - 2t \end{aligned} \qquad t \in \mathbb{R}$$

D3 In order to prove an "if and only if" statement, we need to prove both the "if" and "only if" parts. In this case, this means that we need to prove the statements "if $\vec{x} = \vec{p} + t\vec{d}$ is the equation of a line passing through the origin, then $\vec{p}$ is a scalar multiple of $\vec{d}$" (henceforth notated as $\Rightarrow$) and "$\vec{x} = \vec{p}+t\vec{d}$ is the equation of a line passing through the origin only if $\vec{p}$ is a scalar multiple of $\vec{d}$ (henceforth notated as $\Leftarrow$). The latter statement is equivalent to the statement "if $\vec{p}$ is a scalar multiple of $\vec{d}$, then $\vec{x} = \vec{p} + t\vec{d}$ is the equation of a line passing through the origin." Hopefully you are familiar with the structure of these types of proofs, but if you are not then this is a nice easy one to try. I would have to go into the logical underpinnings of "if and only if" statements to explain any more about why this is the common structure of these types of proofs.

($\Rightarrow$) Suppose $\vec{x} = \vec{p}+t\vec{d}$ is the equation of a line passing through the origin, and let $t_0 \in \mathbb{R}$ be such that $\vec{p} + t_0\vec{d} = \vec{0}$. Then $\vec{p} = -t_0\vec{d}$, so $\vec{p}$ is a scalar multiple of $\vec{d}$.

($\Leftarrow$) Suppose $\vec{p} = s\vec{d}$ for some $s \in \mathbb{R}$. Then $\vec{p} - s\vec{d} = \vec{0}$. As such, $\vec{0}$ is a point on the line $\vec{x} = \vec{p} + t\vec{d}$ (pick $t = -s$), which means that $\vec{x} = \vec{p} + t\vec{d}$ is the equation of a line passing through the origin.