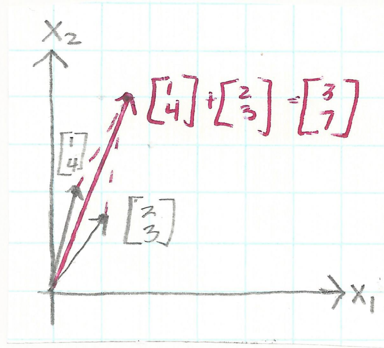
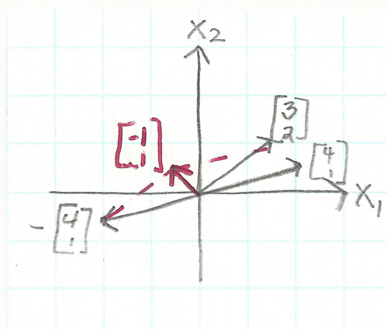
Solution to Assignment 1a

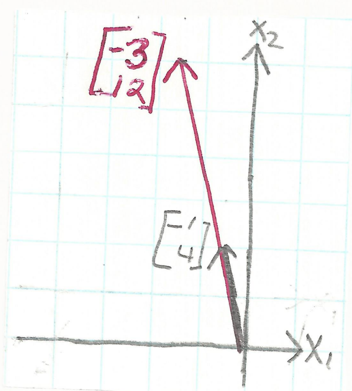
A1(a) $\begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1+2 \\ 4+3 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$



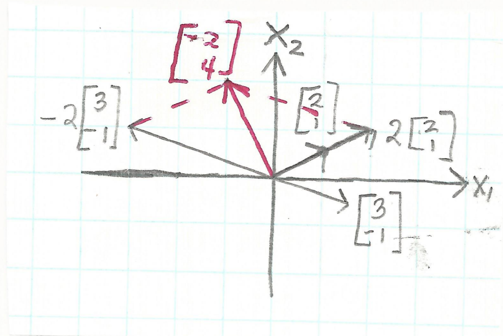
A1(b) $\begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3-4 \\ 2-1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$



A1(c) $3\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} (3)(-1) \\ (3)(4) \end{bmatrix} = \begin{bmatrix} -3 \\ 12 \end{bmatrix}$



A1(d) $2\begin{bmatrix}2\\1\end{bmatrix}-2\begin{bmatrix}3\\-1\end{bmatrix}=\begin{bmatrix}(2)(2)-(2)(3)\\(2)(1)-(2)(-1)\end{bmatrix}=\begin{bmatrix}4-6\\2-(-2)\end{bmatrix}=\begin{bmatrix}-2\\4\end{bmatrix}$



A2(a) $\begin{bmatrix}4\\-2\end{bmatrix}+\begin{bmatrix}-1\\3\end{bmatrix}=\begin{bmatrix}4-1\\-2+3\end{bmatrix}=\begin{bmatrix}3\\1\end{bmatrix}$

A2(b) $\begin{bmatrix}-3\\-4\end{bmatrix}-\begin{bmatrix}-2\\5\end{bmatrix}=\begin{bmatrix}-3-(-2)\\-4-5\end{bmatrix}=\begin{bmatrix}-1\\-9\end{bmatrix}$

A2(c) $-2\begin{bmatrix}3\\-2\end{bmatrix}=\begin{bmatrix}(-2)(3)\\(-2)(-2)\end{bmatrix}=\begin{bmatrix}-6\\4\end{bmatrix}$

A2(d) $\frac{1}{2}\begin{bmatrix}2\\6\end{bmatrix}+\frac{1}{3}\begin{bmatrix}4\\3\end{bmatrix}=\begin{bmatrix}2/2+4/3\\6/2+3/3\end{bmatrix}=\begin{bmatrix}7/3\\4\end{bmatrix}$

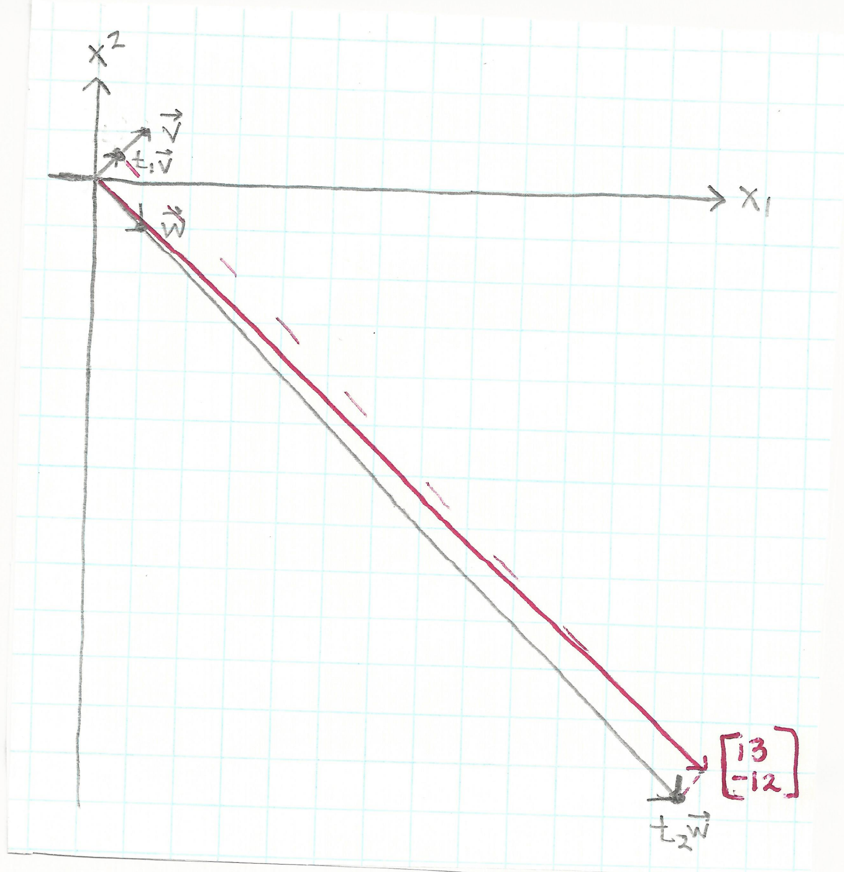
A2(e) $\frac{2}{3}\begin{bmatrix}3\\1\end{bmatrix}-2\begin{bmatrix}1/4\\1/3\end{bmatrix}=\begin{bmatrix}2\\2/3\end{bmatrix}-\begin{bmatrix}1/2\\2/3\end{bmatrix}=\begin{bmatrix}3/2\\0\end{bmatrix}$

A2(f) $\sqrt{2}\begin{bmatrix}\sqrt{2}\\\sqrt{3}\end{bmatrix}+3\begin{bmatrix}1\\\sqrt{6}\end{bmatrix}=\begin{bmatrix}2\\\sqrt{6}\end{bmatrix}+\begin{bmatrix}3\\3\sqrt{6}\end{bmatrix}=\begin{bmatrix}5\\4\sqrt{6}\end{bmatrix}$

D1(a) We need to find real numbers t_1 and t_2 such that $t_1\begin{bmatrix}1\\1\end{bmatrix}+t_2\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}13\\-12\end{bmatrix}$. To do this, I will break the vector equation into its two components, getting the following two equations

$$t_1+t_2=13 \qquad\qquad t_1-t_2=-12$$

Adding the two equations together, we get $2t_1=1$, so $t_1=1/2$. This means that $1/2+t_2=13$, so $t_2=25/2$.



D1(b) We need to find real numbers t_1 and t_2 such that $t_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + t_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. To do this, I will break the vector equation into its two components, getting the following two equations

$$t_1 + t_2 = x_1 \qquad\qquad t_1 - t_2 = x_2$$

Adding the two equations together, we get $2t_1 = x_1 + x_2$, so $t_1 = (x_1 + x_2)/2$. This means that $(x_1 + x_2)/2 + t_2 = x_1$, so $x_1 + x_2 + 2t_2 = 2x_1$, and thus $t_2 = (x_1 - x_2)/2$. (Note that this result agrees with our result for part (a).)

D1(c) We have $t_1 = (x_1 + x_2)/2 = (\sqrt{2} + \pi)/2$ and $t_2 = (\sqrt{2} - \pi)/2$.