CO749 **Assignment 5** **Due: November 30**

Problem 1: Let G be a simple 3-connected graph, let H be a subgraph of G, and let (H_1, H_2) be a separation of H such that:

(i) H is a subdivision of a simple 3-connected graph, and

(ii) each vertex in $V(H_1) \cap V(H_2)$ has degree at least 3 in H.

Prove that, if G_1 is the union of H together with all bridges of (G, H) that have an attachment in $V(H_1) \setminus V(H_2)$, then G_1 is the subdivision of a simple 3-connected graph.

Problem 2: (This fixes a result that I made a mess of in the lectures.) Let $H, G_1, \ldots, G_k$ be subgraphs of a graph G such that

(i) each of $H, G_1, \ldots, G_k$ is a subdivision of a simple 3-connected graph,

(ii) for each $i \in \{1, \ldots, k\}$, the graph G_i is the union of H with some collection of bridges of (G, H),

(iii) $G = G_1 \cup \cdots \cup G_k$, and

(iv) $G_i \cup G_j$ is planar for any $i, j \in \{1, \ldots, k\}$.

Show that G is planar.

Problem 3: Let C be a circuit in a graph G, let $P_1, \ldots, P_k$ be a crooked connector, and let Q_1 and Q_2 be disjoint attachments for $P_1, \ldots, P_k$. Show that either

(i) there is a crooked connector $P'_1, \ldots, P'_k$ with either $E(Q_1) \subseteq E(P'_1 \cup \cdots \cup P'_k) \subseteq E(P_1 \cup \cdots \cup P_k \cup Q_1)$ or $E(Q_2) \subseteq E(P'_1 \cup \cdots \cup P'_k) \subseteq E(P_1 \cup \cdots \cup P_k \cup Q_2)$, or

(ii) there is a crooked connector $P'_1, \ldots, P'_{k+1}$ with $E(Q_1 \cup Q_2) \subseteq E(P'_1 \cup \cdots \cup P'_{k+1}) \subseteq E(P_1 \cup \cdots \cup P_k \cup Q_1 \cup Q_2)$.