SPIN AMPLIFIER FOR SINGLE SPIN MEASUREMENT

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We describe a new approach to the measurement of the state of a collapsed single spin by using many entangled spins as an amplifier. A single target spin is coupled via the natural dipolar Hamiltonian to a large collection of spins. Applying external radio frequency (r.f.) pulses, we can control the evolution of the system so that the ensemble spins reach one of two orthogonal states whose collective properties differ depending on the state of the target spin and are easily measured. We report the result of an experiment simulating this method on an ensemble liquid state NMR Quantum Processor. We suggest therefore that entanglement assisted metrology is compatible with the real control we have over physical spins, since the measurement process can actually be described in terms of the physical Hamiltonian of the spin system. By building on this work, and with the needed technical advances, it should be possible to detect a single nuclear spin.

1. INTRODUCTION.

The measurement of a single nuclear spin is an experimentally challenging task that has stimulated a wide interest since it could bring useful applications as well as valuable physical insights. Single spin measurement, for example, could find applications in spintronics devices[1], a fast evolving field that promises to enhance modern classical computer. It could lead to biomolecular microscopy, which would be a powerful instrument in medical research and structural biology. Finally, it is a critical enabling technology for Quantum Information Processing[2] (QIP), where nuclear spins in solids are candidates to be used as qubits in a quantum computer.

Different techniques are used to attack the problem: force detection[3] and near field optics[4], for example, have reached the limit of single electron spin detection, but they

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have failed to prove the measurement of the quantum state of a single nuclear spin. Instead of exploring new experimental techniques, we try to build a quantum measurement device based on NMR methods, which relies on coherent collective properties of a quantum system. The proposed device, that we call Spin Amplifier, could represent an important step in the road toward single spin detection.

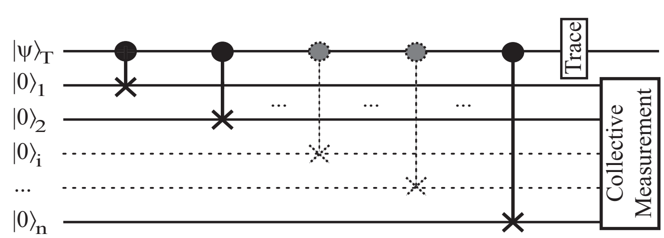
The Spin Amplifier is composed of an ensemble of nuclear spins (about 10^6 spins, which is the current limit for low temperature conventional NMR detection), possibly of a different chemical species than the single spin of interest. The device is put in contact with the spin we want to measure (target spin), to which it is coupled by the usual spin-spin interaction (for example the magnetic dipolar interaction). Because of this coupling, the dynamics imposed to the Amplifier spins can be modulated conditionally on the state of the target spin and the final state of the Amplifier will provide information about the quantum state of the target spin. In its simplest implementation, the interaction of the spin of interest with the ensemble of spins transfers the magnetization properties of the target to the ensemble spins, resulting in the amplification of the target spin signal, up to the point where it is detectable by the conventional inductive means.

This method requires the ability to control a large number of spins by inducing a coherent dynamics, dictated by the interaction with a single spin. Although this is experimentally challenging, we will show that the desired dynamics can actually be described in terms of the physical Hamiltonian of the spin system, and therefore it can be implemented.

2. MEASUREMENT SCHEMES.

To illustrate how a collective measurement of a macroscopic observable can provide the knowledge of the target spin state, we present a simple first scheme (Scheme 1). The ensemble of spins forming the Amplifier is first prepared in the fiducial state: $|00...0\rangle$ (or more generally in a highly polarized state). We assume for simplicity that the target spin has already collapsed into one of the two states $|0\rangle$ or $|1\rangle$. This is not a limitation in the context of QIP, since the measurement of only two orthogonal states is sufficient for the read-out stage in a quantum computation[5].

The desired evolution of the Spin Amplifier is described by a simple quantum circuit consisting of a train of Controlled Not (C-NOT) gates between the target spin and each one of the Amplifier spins.



Scheme 1. Series of C-NOT gates between the target and Amplifier spins. A collective measurement is sufficient to detect the state of the target spin.

The C-NOT gate inverts the state of the controlled qubit if the controlling qubit is in the state $|1\rangle$ and does nothing otherwise. If the controlled spin is in the (known) state $|0\rangle$, this amounts to copying the controlling spin state on the controlled qubit. The "no-cloning" theorem[6], which forbids the creation of identical copies of an arbitrary unknown quantum state, does not apply here, since the target spin is already in one of two orthogonal states. Since the C-NOTs act on the fiducial state $|00...0\rangle_A$, the final state of the Amplifier is $|00...0\rangle_A$ or $|11...1\rangle_A$ if the target spin is in the state $|0\rangle_T$ or $|1\rangle_T$ respectively. At the end of the circuit, the measurement of the Amplifier magnetization along the z-direction, $M_z \propto \langle\psi_A|\Sigma_i\sigma^z{}_i|\psi_A\rangle = \pm M_z(0)$, will indicate the state of the target spin. Even if we address the amplifier spins individually during the scheme, the measurement, on the contrary, involves only collective properties of the Amplifier; the states of the individual spins are not measured.

The C-NOT gate can be easily implemented on a NMR system, with r.f. pulses and periods of evolution under the spin-spin coupling. This first scheme, however, demands a direct coupling between the target spin and each one of the Amplifier spins. Since any spin-spin interaction is local, this requirement is difficult to meet in practice. To avoid invoking direct interactions between the target spin and each ensemble spin, we can use entanglement among the Amplifier spins to develop equivalent schemes relying only on the collective behavior of the system. These schemes are therefore more susceptible to be realizable in the near term because they connect better to the control we have on the physical system.

If the Amplifier spins remain in a factorable state as in scheme 1, the interaction with the target spin can produce only local changes on individual spin states. On the other hand, when the Amplifier is in a macroscopic entangled state (cat-state)[7], a single interaction with the target spin can have a global effect, even if this operation acts on just one spin. For example, we can apply a C-NOT gate between the target spin and its closest neighbor, which is the most strongly coupled to it, after creating a cat-state in the Amplifier. Upon refocusing of the entanglement, we reach two different states depending on the state of the target spin. We can look at the propagator creating the cat-state as if it performed an effective change of basis to a reference frame, where the local C-NOT gate is now a global operator on the Amplifier.

3. EXPERIMENTAL DEMONSTRATION.

We have implemented experimentally a scheme that illustrates the effectiveness of entanglement to propagate a local perturbation (Scheme 2) on a small QIP NMR liquid system. In the experiment, the target spin is the single proton spin of a ^{13}C labeled Alanine molecule ($^2CH_3$-$^1C^T\mathbf{H}(NH_2)$-^3COOH), while the 3 carbons compose the Amplifier. The target spin is therefore represented experimentally by a macroscopic ensemble of spins (the ratio target spins/Amplifier spins is 1:3). Although their state is detectable, it is measured only indirectly, following the scheme proposed.

In a strong external magnetic field, this spin system exhibits a weakly coupled spectrum corresponding to the Hamiltonian:

$$H_{int}= (\omega_1\sigma_1^z+\omega_2\sigma_2^z+\omega_3\sigma_3^z)+\omega_T\sigma_T^z+\pi/2(J_{12}\sigma_1^z\sigma_2^z+J_{13}\sigma_1^z\sigma_3^z+J_{23}\sigma_2^z\sigma_3^z)$$
$$+\pi/2(J_{1T}\sigma_1^z\sigma_T^z+J_{2T}\sigma_2^z\sigma_T^z+J_{3T}\sigma_3^z\sigma_T^z),$$

where the ω's are Larmor frequencies and the J's the spin-spin coupling constants in Hertz. The experiments were carried out on a Bruker AVANCE-300 spectrometer in a field of 7.2 Tesla. The first carbon has a large coupling to the proton, J_{T1}=145.2. Therefore, as the spectrum of the first Carbon at thermal equilibrium shows (Fig. 1), the coupling with the proton are completely resolved and we can separate the signal arising from carbons coupled to protons in the $|1\rangle$ state (left) from the signal due to carbons coupled with $|0\rangle$ state protons (right).

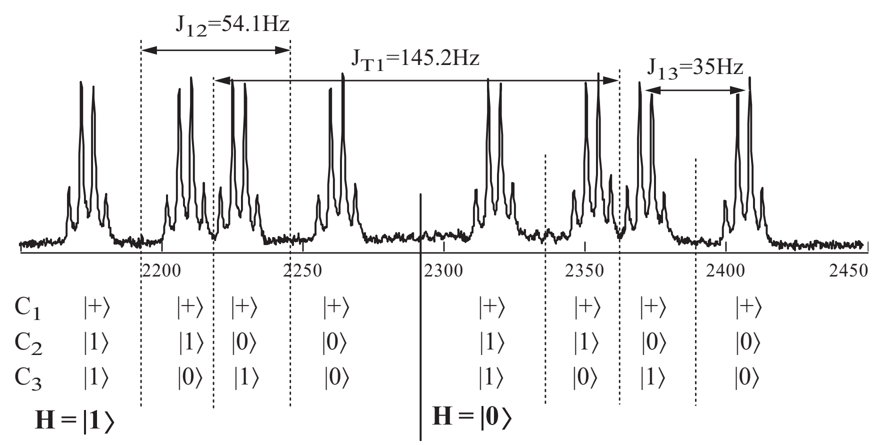
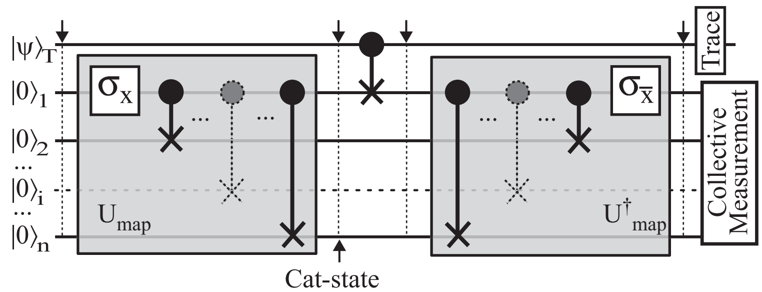


Figure 1. Spectrum of the first carbon at thermal equilibrium, showing the coupling with the proton and the other 2 carbons (The methyl group produces the multiplet splitting).

Before applying the circuit, we first put the proton spin into the identity state and prepare the carbons in the pseudo-pure ground state $|000\rangle$[8,9]. The state of the proton is a mixture of the two decohered states $|0\rangle\langle 0|$ and $|1\rangle\langle 1|$, obtained by simulating a strong measurement with a magnetic field gradient. We can therefore effectively perform two experiments in parallel, and read out the outcomes from just one spectrum, where the lines are separated by the carbon-proton coupling splitting. The pseudo-pure ground state is prepared from the thermal equilibrium state by spatial averaging, which is obtained by creating (by means of magnetic-field gradients) a spatial distribution of states across the ensemble whose mean density matrix is pseudo-pure[10].

Starting from the fiducial state (the pseudo-pure state as in the experiment or more generally the fully polarized state $|00...0\rangle$) the Amplifier is first transformed into the cat-state $(|00...0\rangle - i|11...1\rangle)/\sqrt{2}$ by a $\pi/2$ rotation about σ_x and a series of C-NOTs (see Scheme 2). Then, we invert the state of the first Amplifier spin, conditionally on the state of the target spin, obtaining either $|1\rangle_T(|10...0\rangle - i|01...1\rangle)/\sqrt{2}$ or $|0\rangle_T(|00...0\rangle - i|11...1\rangle)/\sqrt{2}$. When we next apply the inverse transformation to undo the entanglement, what was a local perturbation is propagated through the entire Amplifier. If the target spin is in the $|0\rangle_T$ state, the following C-NOT and σ_x gates bring back the Amplifier to the initial state. Otherwise the C-NOTs produce the state: $|1\rangle_T(|11...1\rangle - i|01...1\rangle)/\sqrt{2}$, which upon the σ_x rotation becomes: $|1\rangle_T|111...1\rangle$. As in the previous scheme, measuring the Amplifier magnetization provides information about the target spin state.



Scheme 2. The creation of an entangled cat-state permits to use only one local action of the target spin. The arrows indicate where diagnostic measurements were performed in the experiment (see Fig. 2)

Diagnostic measurements where performed at four steps in the algorithm illustrated in Scheme 2 (as indicated by arrows). The measurement of the first carbon magnetization is sufficient to determine the state of the 3 Amplifier spins, since we assume that only population elements in the density matrix are nonzero (we would need full tomography to determine the whole density matrix).

The experimental results are shown in Fig. 2, where, as said, the left side of the spectrum corresponds to the case in which the target spin (the proton) is in the $|0\rangle_T$ state and the right part corresponds to the target spin in the $|1\rangle_T$ state. The four spectra illustrate clearly the evolution of the Amplifier spins under the measurement algorithm. The spectrum of the final state of the Amplifier, showing that the polarization of the three carbons has been inverted conditionally on the state of the proton, gives an indirect measurement of its state.

The superiority of scheme 2 on the previous scheme is that we have invoked only one interaction between the target spin and a privileged spin in the Amplifier, a more practical requirement, given the locality of any spin-spin interaction. Extending this scheme to larger systems with a much smaller target to Amplifier spins ratio, to obtain a sensitivity enhancement, is experimentally challenging. Even if we invoked the interaction with the target spin only once, we still needed to act on each Amplifier spin separately, a kind of control that is available only for a limited number of spins.

We can develop other schemes that rely only on collective dynamics and control, if we allow the final states to be just distinguishable, based on their magnetizations, instead of requiring the maximum and minimum magnetization as before. Techniques for introducing entanglement, using only the natural Hamiltonian and r.f. pulses, have been developed in the context of spin counting experiments[11]. The aim of these experiments is to calculate the size of a cluster of interacting spins by measuring its entanglement or more precisely the coherence order of the system (which is the difference in the Hamming weight between two states). The entanglement of a state is related to its coherence order: For example, a cat-state corresponds to an n-spin, n-quantum coherence. To obtain an entangled state therefore, we can use the well established techniques of Multiple Quantum Coherence (MQC)[12,13], and Selective MQC[14] to create the n-quantum coherence operator, $H^{(n)} = \Pi_{k=1}^{n}\sigma_k^{+} + \Pi_{k=1}^{n}\sigma_k^{-}$, which rotates the fully polarized state into the cat-state. In this case, the final magnetization will be either zero or equal to the initial one, but we will have used only the control that is available in conventional NMR.

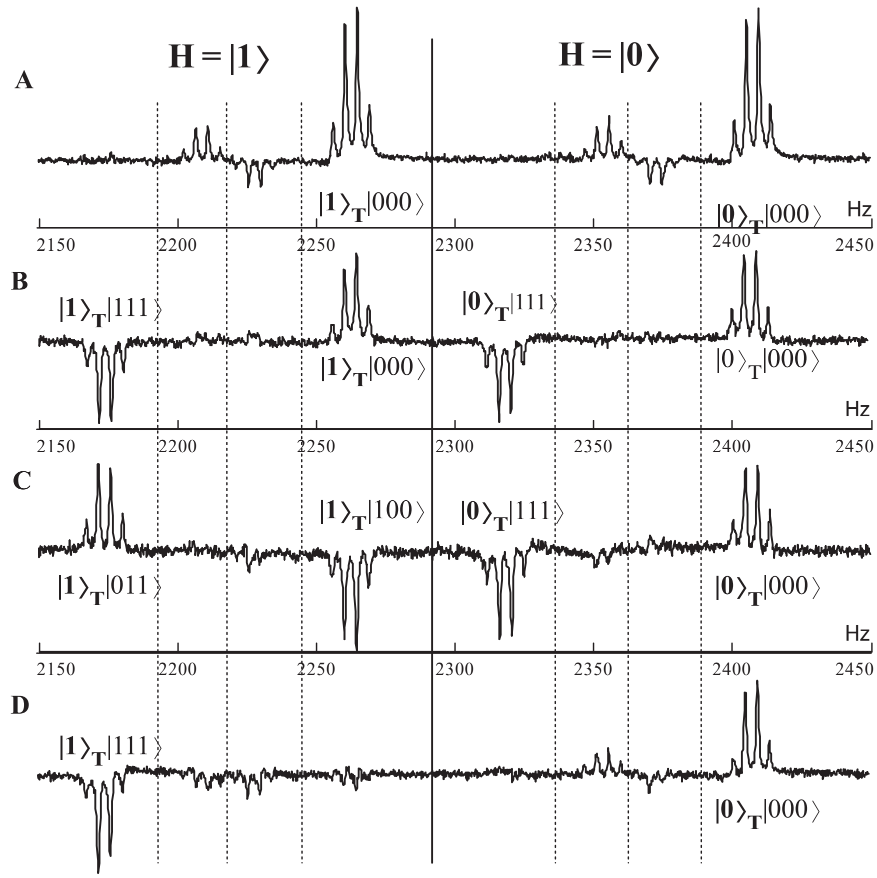


Figure 1. Experimental Results: **A**. Initial State: Pseudo-pure state of the 3 carbons. **B**. Cat-state. **C**. Cat-state, after inverting the state of the 1st carbon conditionally on the state of the proton. **D**. Final state: The polarization of the Carbons coupled to the target spin in state $|1\rangle_T$ (left) has been inverted, while it is unchanged for spins coupled to target spin in the $|0\rangle_T$ state (right).

4. CONCLUSIONS

In conclusion, we have shown that it is possible to transfer polarization from a target spin to an ensemble of spins, through the creation of a highly entangled state with r.f. pulses and the coupling among the target spin and its closest neighbors. This in turn permits us to measure the state of a collapsed single spin, by inferring its state from the collective measurement of the magnetization of a large spin ensemble.

The use of entanglement and multiple pulse sequences allows the scheme to be implemented using only the natural Hamiltonian and the control available in conventional NMR. The methods and physical systems proposed open the possibility to a new class of devices, where quantum effects, such as entanglement, are used to make a transition from microscopic to macroscopic properties.

5. REFERENCES

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