Shaped-Pupil Coronagraphs as High-Contrast Imagers for TPF

An Optimization Success Story

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Large Scale Nonlinear and Semidefinite Programming
Waterloo, Ontario

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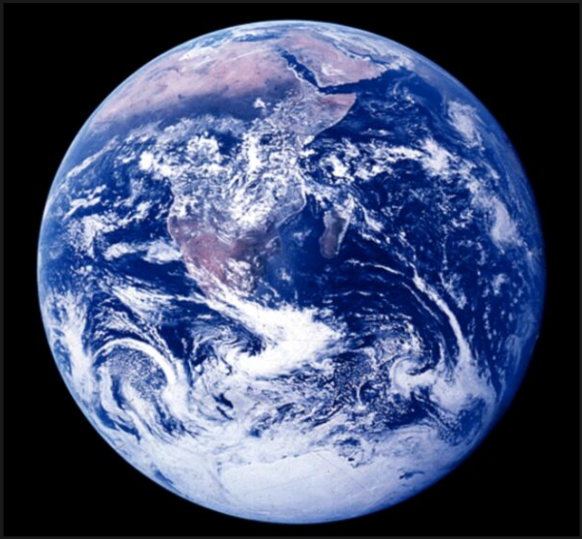
The Big Question: Are We Alone?

- Are there Earth-like planets?
- Are they common?
- Is there life on some of them?



Exosolar Planets—Where We Are Now

There are more than 120 Exosolar planets known today.

They were discovered by detecting a sinusoidal doppler shift in the parent star's spectrum due to gravitationally induced wobble.

This method works best for large Jupiter-sized planets with close-in orbits.

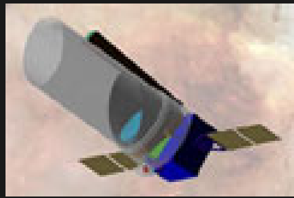
One of these planets, HD209458b, also transits its parent star once every 3.52 days. These transits have been detected photometrically as the star's light flux decreases by about 1.5% during a transit.

Recent transit spectroscopy of HD209458b shows it is a gas giant.

Future Exosolar Planet Missions

- 2006, Kepler a space-based telescope to monitor 100,000 stars simultaneously looking for “transits”.

- 2007, Eclipse a space-based telescope to directly image Jupiter-like planets.



- 2009, Space Interferometry Mission (SIM) will look for astrometric wobble.

- 2014, Darwin is a space-based cluster of 6 telescopes used as an interferometer.
- 2014, Terrestrial Planet Finder (TPF) space-based telescope to directly image Earth-like planets.

Terrestrial Planet Finder Telescope

- DETECT: Search 150-500 nearby (5-15 pc distant) Sun-like stars for Earth-like planets.

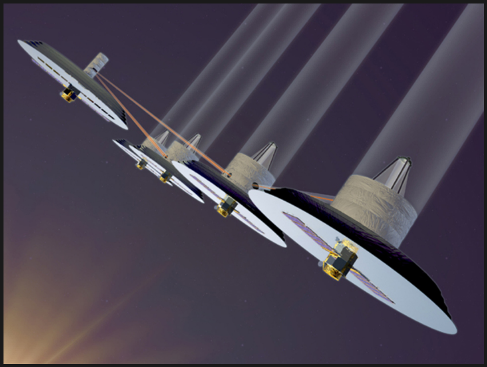
- CHARACTERIZE: Determine basic physical properties and measure "biomarkers", indicators of life or conditions suitable to support it.

Why Is It Hard?

- If the star is Sun-like and the planet is Earth-like, then the reflected visible light from the planet is 10^{-10} times as bright as the star. This is a difference of 25 magnitudes!

- If the star is 10 pc (33 ly) away and the planet is 1 AU from the star, the angular separation is 0.1 arcseconds!

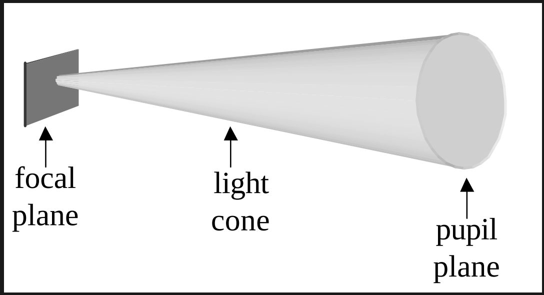
Originally, it was thought that this would require a space-based infrared nulling interferometer (as shown).



However, a more recent idea is to use a single large visible-light telescope with an elliptical mirror (4 m x 10 m) and a *shaped pupil* for diffraction control.

The Shaped Pupil Concept

Consider a telescope. Light enters the front of the telescope—the *pupil plane*.



The telescope focuses the light passing through the pupil plane from a given direction at a certain point on the *focal plane*, say $(0, 0)$.

However, a point source produces not a point image but an *Airy pattern* consisting of an *Airy disk* surrounded by a system of *diffraction rings*.

These diffraction rings are too bright. An Earth-like planet is only about 10^{-10} times as bright as its Sun-like star. The rings would completely hide the planet.

By placing a mask over the pupil, one can control the shape and strength of the diffraction rings. The problem is to find an optimal shape so as to put a very deep *null* very close to the Airy disk.

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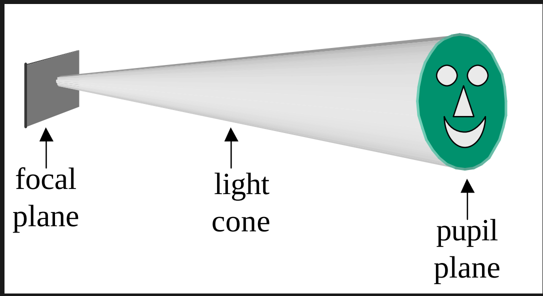

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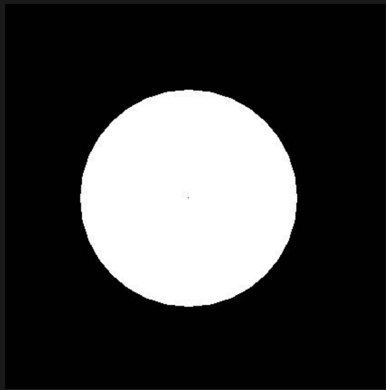
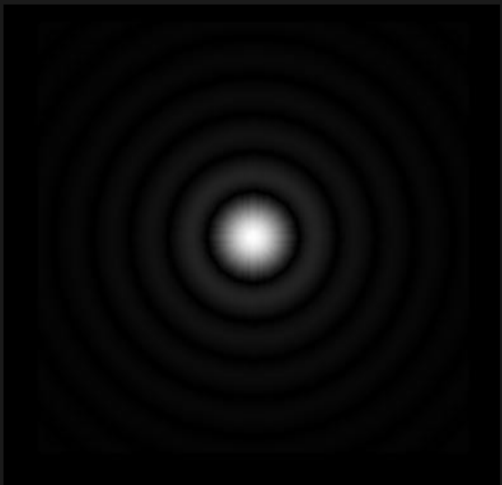
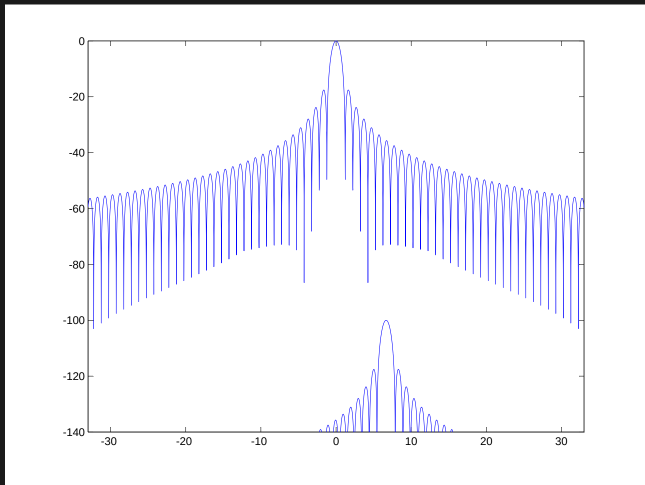
The Airy Pattern







The Princeton Team

David Spergel (Astrophysics, MacArthur genius, Time's astrophysicist of the 21st century, ...)

Jeremy Kasdin (Aerospace Engineering, Gravity Probe-B chief systems engineer)

Robert Vanderbei (the Optimization guy)

Other Princeton members: M. Littman, E. Turner, J. Gunn, M. Carr

And, other members from JPL, Ball Aerospace, and Harvard Center for Astrophysics

Electric Field

The image-plane *electric field* $E()$ produced by an on-axis plane wave and an apodized aperture defined by an *apodization function* $A()$ is given by

$$
\begin{aligned}
E(\xi, \zeta) &= \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} e^{-2\pi i (x\xi + y\zeta)} A(x,y)\, dy\, dx \\
&\vdots \\
E(\rho) &= 2\pi \int_0^{1/2} J_0(2\pi r \rho) A(r) r\, dr,
\end{aligned}
$$

where J_0 denotes the 0-th order Bessel function of the first kind.

The unitless pupil-plane "length" r is given as a multiple of the aperture D.

The unitless image-plane "length" ρ is given as a multiple of focal-length times wavelength over aperture ($f\lambda/D$) or, equivalently, as an angular measure on the sky, in which case it is a multiple of just λ/D. (Example: $\lambda = 0.5\mu$m and $D = 10$m implies $\lambda/D = 10$mas.)

The *intensity* is the square of the electric field.

Performance Metrics

Inner and Outer Working Angles

$$\rho_{\mathrm{iwa}} \qquad \rho_{\mathrm{owa}}$$

Contrast:

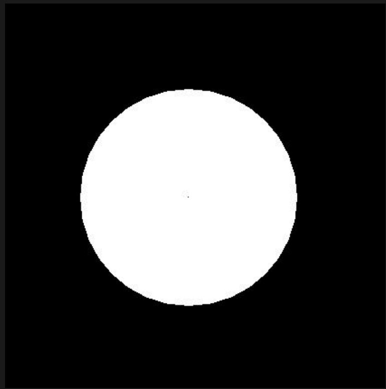
$$E^2(\rho)/E^2(0)$$

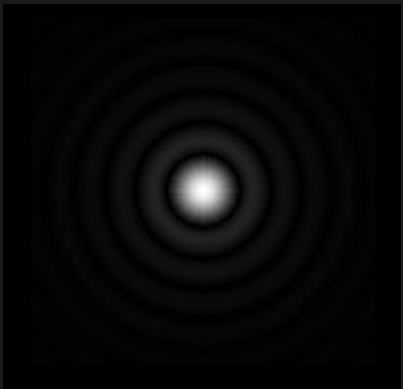
Airy Throughput:

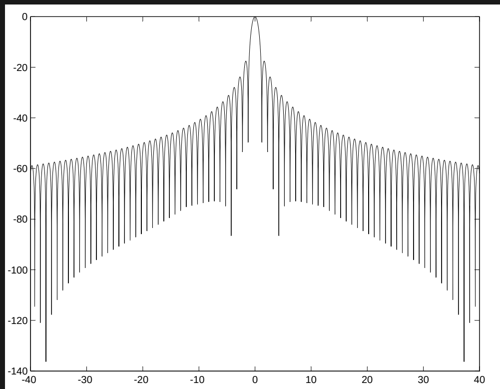
$$\frac{\int_0^{\rho_{\mathrm{iwa}}} E^2(\rho) 2\pi\rho d\rho}{(\pi(1/2)^2)} = 8 \int_0^{\rho_{\mathrm{iwa}}} E^2(\rho)\rho d\rho.$$

Clear Aperture—Airy Pattern

$$\rho_{\text{iwa}} = 500 \quad \mathcal{T}_{\text{Airy}} = 84.2\%$$







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Optimization

Find *apodization* function $A()$ that solves:

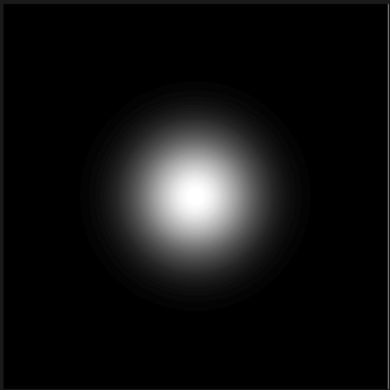
$$\begin{aligned} \text{maximize} \quad & \int_0^{1/2} A(r) 2\pi r dr \\ \text{subject to} \quad & -10^{-5} E(0) \le E(\rho) \le 10^{-5} E(0), \qquad \rho_{\text{iwa}} \le \rho \le \rho_{\text{owa}}, \\ & 0 \le A(r) \le 1, \qquad 0 \le r \le 1/2, \end{aligned}$$

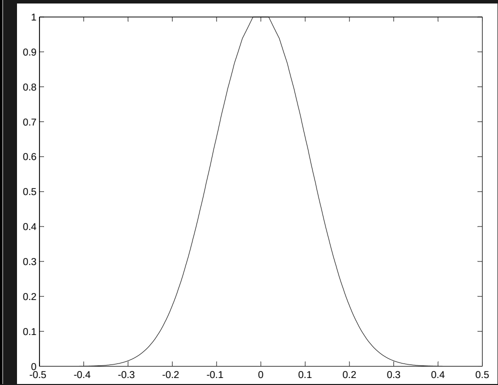
Note the similarity to FIR filter design and antenna array design problems (see Boyd's talk next).

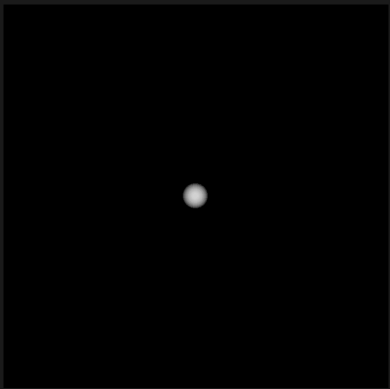
Apodization

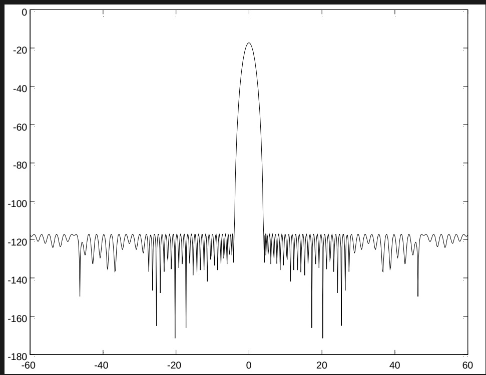
$$\rho_{\text{iwa}} = 4 \quad \mathcal{T}_{\text{Airy}} = 9\%$$

Excellent dark zone. Unmanufacturable.









Masks

Consider a binary apodization (i.e., a mask) consisting of an opening given by

$$A(x, y) = \begin{cases} 1 & |y| \leq a(x) \\ 0 & \text{else} \end{cases}$$

We only consider masks that are symmetric with respect to both the x and y axes. Hence, the function $a()$ is a nonnegative even function.

In such a situation, the electric field $E(\xi, \zeta)$ is given by

$$\begin{aligned} E(\xi, \zeta) &= \int_{-\frac{1}{2}}^{\frac{1}{2}} \int_{-a(x)}^{a(x)} e^{i(x\xi + y\zeta)} dy dx \\ &= 4 \sum_j \int_0^{\frac{1}{2}} \cos(x\xi) \frac{\sin(a(x)\zeta)}{\zeta} dx \end{aligned}$$

Maximizing Throughput

Because of the symmetry, we only need to optimize in the first quadrant:

$$\text{maximize } 4\int_0^{\frac{1}{2}} a(x)dx$$

$$\begin{aligned} \text{subject to } & -10^{-5}E(0,0) \le E(\xi,\zeta) \le 10^{-5}E(0,0), && \text{for } (\xi,\zeta)\in\mathcal{O} \\ & 0 \le a(x) \le 1/2, && \text{for } 0 \le x \le 1/2 \end{aligned}$$

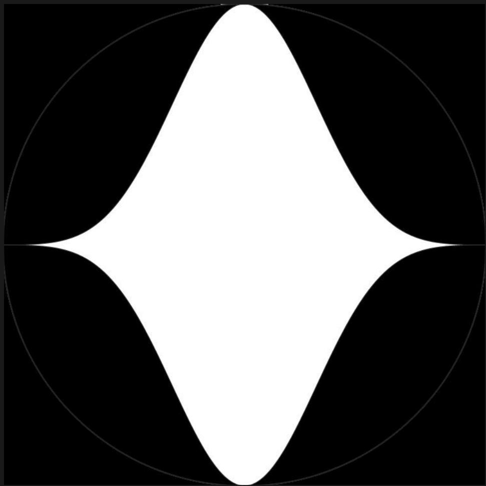
The objective function is the total open area of the mask. The first constraint guarantees 10^{-10} light intensity throughout a desired region of the focal plane, and the remaining constraint ensures that the mask is really a mask.

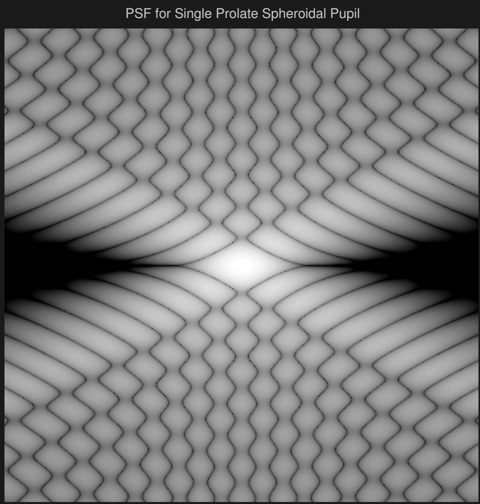
If the set $\mathcal{O}$ is a subset of the x-axis, then the problem is an infinite dimensional linear programming problem.

One Pupil w/ On-Axis Constraints

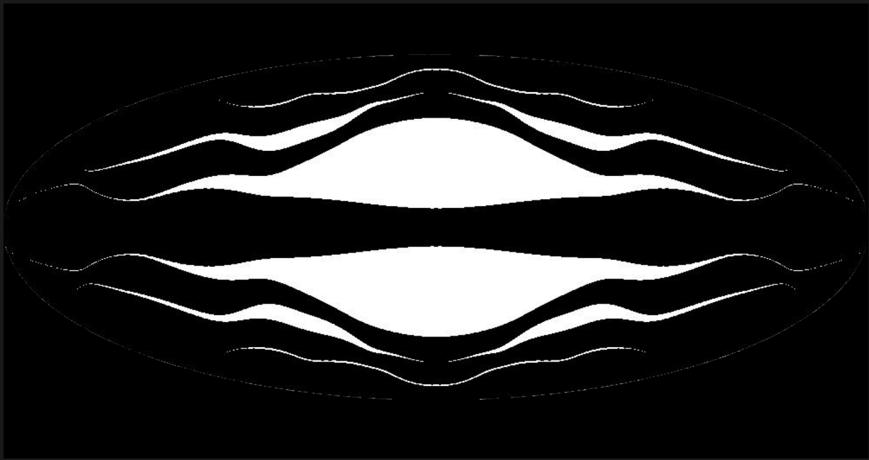
$$\rho_{\text{iwa}} = 4 \quad \mathcal{T}_{\text{Airy}} = 43\%$$

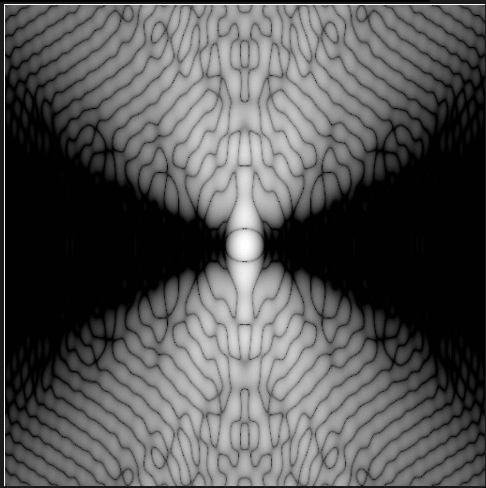
Small dark zone...Many rotations required





Multiple Pupil Mask





$\rho_{\text{iwa}} = 4$

$\mathcal{T}_{\text{Airy}} = 30\%$

Throughput relative to ellipse

11% central obstr.

Easy to make

Very few rotations

Concentric Ring Masks

Recall that for circularly symmetric apodizations

$$E(\rho) = 2\pi \int_0^{1/2} J_0(2\pi r \rho) A(r) r dr,$$

where J_0 denotes the 0-th order Bessel function of the first kind.

Let

$$A(r) = \begin{cases} 1 & r_{2j} \le r \le r_{2j+1}, \qquad j = 0, 1, \ldots, m-1 \\ 0 & \text{otherwise,} \end{cases}$$

where

$$0 \le r_0 \le r_1 \le \cdots \le r_{2m-1} \le 1/2.$$

The integral can now be written as a sum of integrals and each of these integrals can be explicitly integrated to get:

$$E(\rho) = \sum_{j=0}^{m-1} \frac{1}{\rho} \left(r_{2j+1} J_1 \left(2\pi \rho r_{2j+1} \right) - r_{2j} J_1 \left(2\pi \rho r_{2j} \right) \right).$$

Optimization Problem

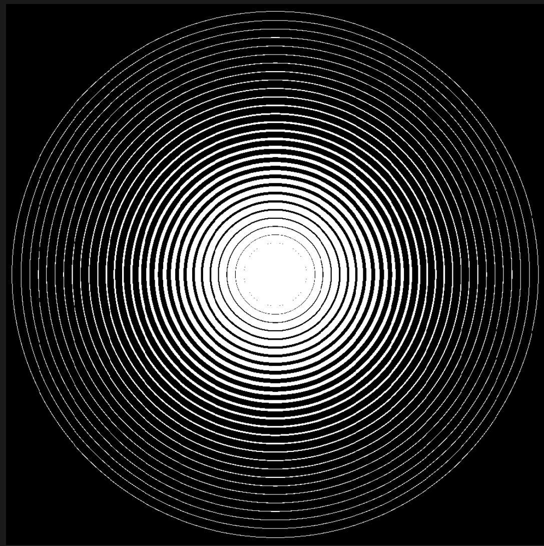
$$\text{maximize} \sum_{j=0}^{m-1} \pi (r_{2j+1}^2 - r_{2j}^2)$$

$$\text{subject to:} \quad -10^{-5} E(0) \leq E(\rho) \leq 10^{-5} E(0), \quad \text{for } \rho_0 \leq \rho \leq \rho_1$$

where $E(\rho)$ is the function of the r_j's given on the previous slide.

This problem is a semiinfinite nonconvex optimization problem.

Concentric Ring Mask

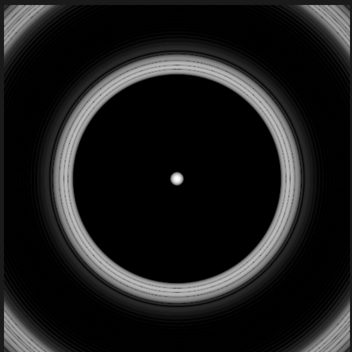


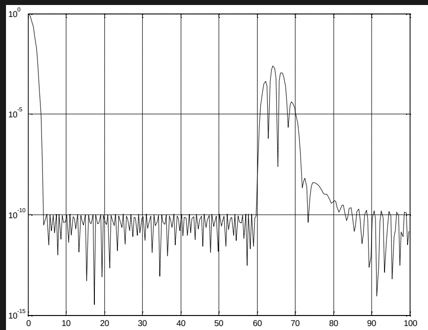
$\rho_{\text{iwa}} = 4 \quad \rho_{\text{owa}} = 60$

$\mathcal{T}_{\text{Airy}} = 9\%$

Lay it on glass?

No rotations





Optimization Success Story

From an April 12, 2004, letter from Charles Beichman:

Dear TPF-SWG,

I am writing to inform you of exciting new developments for TPF. As part of the Presidents new vision for NASA, the agency has been directed by the President to *conduct advanced telescope searches for Earth-like planets and habitable environments around other stars.* Dan Coulter, Mike Devirian, and I have been working with NASA Headquarters (Lia LaPiana, our program executive; Zlatan Tsvetanov, our program scientist; and Anne Kinney) to incorporate TPF into the new NASA vision. The result of these deliberations has resulted in the following plan for TPF:

1. Reduce the number of architectures under study from four to two: **(a) the moderate sized coronagraph, nominally the 4x6 m version now under study**; and (b) the formation flying interferometer presently being investigated with ESA. Studies of the other two options, the large, 10-12 m, coronagraph and the structurally connected interferometer, would be documented and brought to a rapid close.

2. Pursue an approach that would result in the launch of BOTH systems within the next 10-15 years. The primary reason for carrying out two missions is the power of observations at IR and visible wavelength regions to determine the properties of detected planets and to make a reliable and robust determination of habitability and the presence of life.

3. **Carry out a modest-sized coronagraphic mission, TPF-C, to be launched around 2014**, to be followed by a formation-flying interferometer, TPF-I, to be conducted jointly with ESA and launched by the end of decade (2020). This ordering of missions is, of course, subject to the readiness of critical technologies and availability of funding. But in the estimation of NASA HQ and the project, the science, the technology, the political will, and the budgetary resources are in place to support this plan.

...

The opportunity to move TPF forward as part of the new NASA vision has called for these rapid and dramatic actions. What has made these steps possible has been the hard work by the entire team, including the TPF-SWG, the two *TPF architecture teams*, and all the technologists at JPL and around the country, which has demonstrated that NASA is ready to proceed with both TPF-C and TPF-I and that the data from these two missions are critical to the success of the goals of TPF. We will be making more information available as soon as additional details become available. Thank you for all your help in preparing TPF to take advantage of this opportunity.

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