Referee Report on

"The Q Method for Symmetric Cone Programming"

by Y. Xia and F. Alizadeh

Summary

In this paper, the Q method for symmetric cone programming is considered. After a long introduction on Euclidean Jordan algebra and symmetric cones, they proposed the Q method for symmetric cone programming and proved non-singularity of the Newton system at optimum under some assumptions. Furthermore in Section 4, they proposed an infeasible interior-point algorithm using the Q method and prove global convergence of a variant of the algorithm. Finally, results of numerical experiments are shown.

Major Comments

1. I have a doubt on the assumption of one of the main results Lemma 3.3. In Lemma 3.3, it is assumed that all block components of the optimal solutions $\boldsymbol{x}_i$ and $\boldsymbol{z}_i$ are regular. But can it be possible that all of them are regular at optimal solutions? Consider, for example, the real symmetric matrix case. If a symmetric matrix has a 0 eigenvalue, then it is not regular. We know that $\boldsymbol{x}_i$ or $\boldsymbol{z}_i$ must have a 0 eigenvalue at optimum. I'm confused at this point.

2. Global convergence of the algorithm is interesting, but this section is poorly written and I could not understand the proof completely. In particular, the author must provide us the assumptions needed before the analysis starts. I list up some of the questions of the proof later.

3. The numerical experiments section is also poorly written and quite confusing. First, there is no explanation about the algorithm used. Second, averages of the initial infeasibility are shown in the table, but not those of the final one. So, in fact only the average number of iterations are shown on the table. I do not understand what the author is trying to convey through this experiment. Third, it seems that the types of SOCP blocks (boundary, interior, or zero at the optimum) are fixed through all the randomly generated problems. Again I don't understand the reason.

4. In fact, Figure 1 (the results of numerical experiments) is identical to the table on page 21 of [*]. The authors must explain the relationship between the current paper and [*].

   [*] The Q Method for Second-order Cone Programming, F. Alizadeh and Y. Xia, McMaster University, AdvOl-Report No. 2004/15, October 2004, Hamilton.

The Q method was originally proposed for SDP [3, 5] by the group including one of the authors of this paper, and then for SOCP [*] by the authors of the current paper. It seems that the Q method is neither actively investigated in theory nor used in practice. In fact, I don't know any papers on the Q method other than [3, 5, *]. Naturally, I doubt whether the Q method is important in any sense. The authors must provide a definite evidence for the (potential) power of the Q method, if they want to publish this paper on a top journal.

The non-singularity of the Newton system at optimum is not enough for this purpose. The global convergence result is interesting and may be publishable, but I'm not sure whether Mathematical Programming is the right place even if the result is correct, because of the strange regularity assumption. The numerical experiments in this paper are inadequate.

My impression on this paper is negative. At least, the paper should be re-submitted after a major revision and reviewed again.

Minor Comments

1. Section 2 is too long. It seems that there are many unnecessary materials. For example, Theorem 2.6 is not necessary. Choose what is needed for this paper.

2. Page 22, Lemma 3.3. What is (x, z) regularity?

3. Page 25, just after the description of the algorithm: Explain why $\tilde{\alpha} > 0$.

4. Page 28, By the same arguments as those in [15]: Please write the arguments. The arguments show that the complementarity is reduced at each iteration, which is very important. The reader wants to know the reason immediately.

5. Section 5. This section should be completely rewritten. In particular, problems having both SDP and SOCP cones must be solved, because this paper is about symmetric cone programming. Also explain in the text what is shown by the experiments. If you want to insist the algorithm can get high accuracy, you should compare it with the standard primal dual interior point method.