Warm Startable Interior Point Method (WS-IPM)

Jos F. Sturm

CentER, Tilburg University, The Netherlands

`http://fewcal.uvt.nl/sturm`

ISMP Copenhagen, August 2003

Overview

- Master and Subproblem
- Search Direction
- WS-IPM
- Numerical Results

Goldman-Tucker Self-Dual Model

$$
\begin{array}{ll}
\text{Find} & x + z > 0 \\
\text{s.t.} & Mx + z = 0 \\
 & x, z \geq 0,
\end{array}
$$

where $M + M^{\mathsf{T}} = 0$.

Master Problem

$$\begin{aligned} \text{min} \quad & y_0 \\ \text{s.t.} \quad & Mx + z = y_0 r \\ & r^{\mathsf{T}} x + s_0 = 1 \\ & x, z, y_0, s_0 \geq 0 \end{aligned}$$

NB: $y_0 = y_0 s_0 + x^{\mathsf{T}} z$.

Path-Following Direction

$$p_0 = \frac{\gamma\mu}{s_0} - y_0$$

$$p_i = \frac{\gamma\mu}{z_i} - x_i \text{ for } i = 1, \ldots, n$$

where

$$\mu = \frac{y_0}{n+1}$$

Sub-Problem

SP has only (x_i, z_i) for $i \in B$.

Direction $(\Delta y_0, \Delta x_B, \Delta z_B)$.

$$\begin{cases} \Delta y_0 = s_0 p_0 + z^\mathsf{T} p \\ \Delta x_i + (x_i/z_i)\Delta z_i = p_i \text{ for } i \in B \\ M_{BB}\Delta x_B + \Delta z_B = \Delta y_0 r_{SP} \end{cases}$$

Extend Direction to MP

$$\Delta x_N = \frac{\Delta y_0}{y_0} x_N$$

$$\Delta z_N = \frac{\Delta y_0}{y_0} z_N + M_{NB}(\frac{\Delta y_0}{y_0} x_B - \Delta x_B)$$

$$\Delta s_0 = (1 - s_0)\frac{\Delta y_0}{y_0} - \frac{z^\top \Delta x + x^\top \Delta z}{y_0}$$

Evaluate MP Direction

Let t^* maximal feasible step-length in SP.

- $y_0 + t^* \Delta y_0 \geq 0$

- $x + t^* \Delta x \geq 0$

- If $z_i + t^* \Delta z_i < 0$ then add cut to SP

- If $s_0 + t^* \Delta s_0 < 0$ then cheap progress! (Along $(-y_0, -x, -z)$)

Generate Search Direction

Step 1 Compute $(\Delta y_0, \Delta x_B, \Delta z_B)$ and t^* in SP.

Step 2 Extend to $(\Delta x, \Delta z)$ in MP.

Step 3 If $z_i + t^* \Delta z_i < 0$ $\exists i$ then let

$$B := B \cup \{i\}$$

and return to Step 1. Otherwise, STOP.

WS-IPM

Step 0 Initialize (y_0, s_0, x, z) and B.

Step 1 If residual $< \epsilon$ then STOP

Step 2 Generate search direction (see previous slide).

Step 3 Take step

Step 4 Guess

$$B = \{i \mid \frac{\Delta x_i}{x_i} - \frac{\Delta z_i}{z_i} > 0\}$$

Step 5 Return to Step 1.

Conclusion

- WS-IPM Step generated from SP. Matrix in SP roughly $m \times m$ size.

- SP-updates are like Simplex iterations

- Main WS-IPM steps often more effective than standard IPM steps, but potentially much cheaper

- More effective B-guess and/or update rule needed for early iterates (after cold-start).

- Serious warm-start evaluation needed, e.g. in B&B for MIP.