Referee's Report on
Zhaosong Lu and R.D.C. Monteiro "A modified nearly exact method for solving low-rank trust region subproblem"

The main result of this paper is proposing a modified Moré-Sorensen method for solving the trust region subproblem when the Hessian of the objective quadratic function is the sum of a diagonal matrix and a low-rank matrix. The paper also presented the numerical results based on their modified log-barrier method, and made comparison with the LANCELOT method.

The paper explores the special structure of the "diagonal plus low rank", and makes good uses of the special structure to reduce the computation costs of the original Moré-Sorensen method. Therefore for the special case when the Hessian matrix has the "diagonal plus low rank" form (which happens when the Hessian is generated by limited memory quasi-Newton updates), the modified method improve computing efficiency of the original Moré-Sorensen method by reducing computation operations.

Therefore it seems to me that the importance of this paper is its clever structure usages and practical implementation of the Moré-Sorensen method instead of theoretical results.

One of the most major improvement/modification is the handing of the hard case termination. This paper makes a nice observation and show that the null space vector u_λ can be computed by working on a low-rank matrix $V_2^T H(\lambda)^{-1} V_2$. However, we will have no hard-case if the Hessian is positive definite(particularly when the matrix is generated by L-BFGS).

Related earlier work is as follows. The standard trust region subproblem is actually equivalent to a low dimensional trust region subproblem if the Hessian is generated by L-BFGS method (for example, see Wang, Wen and Yuan(2004); Yuan(2004)).

Z.H. Wang, Z.W. Wen, and Y. Yuan (2004): "A subspace trust region method for large scale unconstrained optimization", in: Y.Yuan, ed. Numerical Linear Algebra and Optimization(Proceeding of the 2003 International Conference on Numerical Optimization and Linear Algebra) (Science Press, Beijing/NewYork, 2004) pp. 265-274.

Y. Yuan(2004): "A Subspace Trust Region Algorithm", presented at the Conference on Multiscale Optimization Methods and Applications(Center for Applied Optimization, University of Florida, February 26-28, 2004).