Summary

This paper introduces a modification of the nearly-exact method of Moré and Sorensen for solving the trust-region subproblem (TRS) when the matrix in the quadratic objective is the sum of a diagonal matrix and a low-rank update. The authors consider large-scale constrained optimization problems with bound constraints and a small number of linear constraints. The above TRS occur when a log-barrier approach is applied and a trust-region algorithm is used to solve each log-barrier subproblems. The paper is divided mainly in 3 parts. In the first part, the authors recall the algorithm of Moré and Sorensen with special emphasis on the steps where one needs complete or partial Cholesky factorizations. In the second part, the authors present a modification of the algorithm of Moré and Sorensen where they exploit the low-rank structure in any step that requires a Cholesky factorization in the latter algorithm. In particular, this is true for the Newton iteration on the secular function. Most computations rely on the Sherman-Morrison formula. In the third part, the authors test their log-barrier approach against LANCELOT on bound constrained and unconstrained optimization problems from the CUTEr package. In general, the approach shows improvement both in terms of the quality of the optimal solution and the speed of computation.

Comments

The paper is well structured and the results easy to follow. The motivation for low-rank TRS is well stated initially and we know throughout the paper where the authors are taking us. I have a few minor comments and typos to mention:

1. pp. 2 L20: Two comments here. First, I believe the method of Steihaug was also introduced independently by Toint [3]. Second, maybe one should add as a reference the GLTR method of Gould and al. [2] which builds on the Steihaug method using the Moré and Sorensen algorithm on tridiagonal TRS. It can yield surprisingly good approximate solution with a few extra computing efforts.

2. pp. 3 L19 λ_{min}) should probably be $\lambda_{min}(E)$.

3. pp. 4 L8: Since the authors refer often to the LRTR subproblem in the paper, it might be helpful to label it explicitly here.

4. 7 L7 p should be $p(\lambda)$ here or mention $p = p(\lambda)$ when the argument is omitted.

5. pp. 20 For problem FLETCBV2, what does it mean for the number of iterations to be 0? Was the initial approximation optimal?

6. pp. 24 Reference [11] is now published, see the bibliography below, reference [1].

Finally I believe there is significant contributions in this paper and recommend its publication.

References

[1] C. Fortin and H. Wolkowicz. The trust region subproblem and semidefinite programming. *Optimization Methods and Software*, 19(1):41–67, February 2004.

[2] N.I.M. Gould, S. Lucidi, M. Roma, and P.L. Toint. Solving the trust-region subproblem using the lanczos method. *SIAM Journal on Optimization*, 9(2):504–525, 1999.

[3] P.L. Toint. Towards an efficient sparsity exploiting newton method for minimization. In Iain S. Duff, editor, *Sparse matrices and their uses*, Institute of Mathematics and its Applications Conference Series, pages xii+387, London, 1981. Academic Press Inc. [Harcourt Brace Jovanovich Publishers].