Letter to the referees

On the solution of large-scale SDP problems by the modified barrier method using iterative solvers

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We would like to thank all three referees for careful reading of the manuscript and many valuable comments. We believe that we have addressed all of them in the revised version. The main changes to the original version are summarized below:

A. Based on the observation of one of the referees, we have implemented a new version of the "Hessian free" code in which we use an implicit formula for calculation of the Hessian vector product. This version of the code (called PEN-I-PCG in the paper) turned out to be the best tool for the large-n-small-m problems. We have changed all the tables accordingly. Still, we opted for keeping the version with explicit Hessian calculation in (some) tables, so that the reader can get a complete picture of various approaches.

B. We have changed the stopping criteria in the codes in order to see clearly what are the algorithms giving up and to improve the quality of the solution. (In the original version, the most critical point was often the primal feasibility of the solution.) With the improved criteria, we have recomputed all the examples. As a result, the codes now need more CG steps in certain examples at a price of guaranteed quality of the approximate solutions. Also the number of CG steps grows faster (compared to the original version), when higher accuracy is required (see the last paragraph). Still the overall conclusions remain valid.

C. We skipped completely the paragraph on the MON preconditioner. After it was fully implemented in the code, it turned out (unfortunately) that it was not competitive. The drawback is the high cost of computation of the smallest eigenvalue of the Newton matrix and often many update steps needed to form the preconditioner (which, as such, is then very efficient but overall very costly).

Answers to specific point:

Referee 1 (*The paper is on...*)

1. We tried to answer all the points in the revised version. See also point B. above.

2a. In our opinion, the theory of inexact Krylov methods does not fully apply here: the theory assumes that

– the higher precision of the matrix-vector product is required, the more costly it is (e.g., the matrix elements are computed by another iterative process);

– the required accuracy of the solution is relatively high.

The general guidelines of such method say that one should start with a high accuracy of the matrix-vector product and this accuracy can be relaxed, as the computed residuum becomes smaller and smaller.

In our situation, the cost of the finite difference formula is not influenced by the size of h. We try to have the formula as accurate as possible; however, once ε is too small, the accuracy is influenced by round-off errors and, eventually, the formula becomes useless for very small ε. Hence the choice of ε is a compromise between the the accuracy of a finite-difference formula and influence of round-off errors. Further, the required accuracy of the computed residua is relatively very small in our case; in this case, according to the guidelines, we anyway would have to start the process with a relatively high accuracy of the matrix-vector product and then, before we had a chance to relax this accuracy, the process would be stopped.

We added a comment to the respective paragraph trying to explain this.

3a+c. We put less weight on the diagonal and SGS preconditioners and more on the L-BFGS one. We removed the AINV preconditioner. We chose not to remove the diag and SGS in order to give the reader an idea of the behavior of known algorithms and their comparison to the most used one. Also, the L-BFGS is not always preferable (see the TRUSS collection).

See also point C. above.

3b. Included in the revised version.

4. See point A. above.

Referee 2 (*The paper deals with...*)

3. This has been explained in the revised version, referring to the literature and also to the similarity with the Toint-Steihaug method.

7. See point C. above.

All the other points have been corrected in the revised version.

Referee 3 (*The paper continues...*)

1.-6. The comments have been addressed in the revised version.

7. We are grateful to the referee for pointing out this possibility. We have designed a new version of the code (called PEN-I-PCG) based on the proposed formula and incorporated it in the paper and numerical tests. It turns out that this new version outperforms the approximate version of the code.

8. Corrected in the revised version.

9. AINV preconditioner was removed.

11. Explained and modified in the revised version.

12. SDPLR was run in the default mode, just the stopping criterion was modified to match those of PENNON. We believe that any modification with respect to the structure of a specific problem would be a bit unfair in comparison with a general purpose solver. That is also the reason why we do not mention bundle codes at all (though they are for certain problems by far the fastest ones).

13. We have added two more problems from the HM collection.