

Response to Comments of Referee #1
“Behavioral Measures and their Correlation with IPM Iteration Counts on
Semi-Definite Programming Problems” by R.M. Freund, F. Ordóñez, and
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We are well aware that the performance of IPM algorithms on problem instances is mostly determined by the actual heuristic implementation details in the algorithm. However, what we set out to investigate in this work is whether certain behavioral measures of the *problem instances themselves* are correlated with the performance of state-of-the-art IPM codes for SDP. The fact that such correlations exist (as manifested by the high correlations found between behavioral measures and IPM iterations) is surprising if we note that we are *not* considering important features of the algorithm and that these behavioral measures (geometric and condition number) originally arose in a different context – namely the complexity of theoretical algorithms – which are quite different from what is implemented in efficient software.

The idea of this paper is to test which of the four proposed behavioral measures related to problem instances (geometry measure g^m , condition number $C(d)$, non-strict complementarity κ , and degeneracy γ) might be related to the inherent ease or difficulty of a problem instance’s solution by state-of-the-art IPM software. All four behavioral measures are related with the geometric notions of the primal and dual feasible regions and/or optimal solutions, and we have reason (and now evidence) to believe that they are capable of identifying what impacts the performance of most current IPM solvers.

For illustration we present below the correlations obtained between the measures investigated in our paper and the number of IPM iterations for 5 different SDP solvers. We used the IPM iterations reported in Hans Mittleman’s benchmarks of Jan 31, 2005 (available at <http://plato.asu.edu/bench.html>) and were able to obtain 26 different problems from the SDPLIB suite for which we have computed measures and have iteration counts for the solvers CSDP, DSDP, SDPA, SDPT3 and SEDUMI. We also consider the iteration counts obtained with SDPT-3.1 in our paper. Here are the sample correlations between IPM iterations for the different solvers and the four measures considered in our paper when both quantities are finite numbers:

	$\log(g^m)$ (21)	$\log(C(d))$ (16)	κ (26)	γ (22)
CSDP	0.883	0.473	0.331	0.029
SDPA	0.860	0.597	0.308	-0.142
SEDUMI	0.911	0.537	0.189	-0.129
SDPT3	0.850	0.417	0.352	0.056
SDPT3-3.1	0.863	0.410	0.444	-0.038
DSDP	0.334	0.387	-0.184	-0.068

The number of problems used to calculate each correlation is noted at the top of the column by a bold number.

All of the solvers in this table use a primal-dual path-following framework except for DSDP, which instead uses a dual potential reduction framework. We note that although there is some variation across solvers, from these preliminary results we notice that the most significant correlations are obtained for $\log(g^m)$ and $\log(C(d))$ across all of the solvers that use the primal-dual path-following framework. In addition we notice that SDPT-3.1 is a representative solver from this group.