# REFEREE REPORT ON: BAHAVIORAL MEASURES AND THEIR CORRELATION WITH IPM ITERATION COUNTS ON SEMIDEFINITE PROGRAMMING PROBLEMS. BY ROBERT FREUND, FERNANDO ORDOÑEZ AND KIM CHAUN TOH.

This paper shows interesting computational results regarding the potential explanatory value of different condition measures for SDP problems and its actual computational difficulty. The work is a natural extension of a previous work by F. Ordoñez and R. Freund in which they analyze the computational behavior of a commercial interior point implementation, and its connection to some condition measures. In the current work the authors follow the same general approach, but they have basically extended the condition measures to the setup of Semidefinite Programming.

The paper is structured in basically two parts: a development of the different condition measures which will be used, and a second part devoted to the computational results and conclusions. A large appendix include detailed numerical results.

I found the work well written (except for some observations which I have made later) and correct although, in my opinion, it is not as important as the results from the first work of the authors. That work was the first attempt to seriously study the empirical connections between conditioning and actual performance of algorithms. The current work does the same for SDP. However, there is also some important contributions in the current work regarding the definition of condition measures for SDP and the fact that the same pattern of behavior repeats is an indication of a deeper connection between geometric condition measures and the performance of IPM.

I do have, however, a couple of major concerns about the current version of the paper. First, the authors sustain that a significant correlation is obtained between the computational performance and the aggregate condition measure $g^m$. An acceptable correlation is obtained with $C(d)$, the Renegar's condition measure. Almost no correlation is detected with degeneracy measures and non strict complementarity. Given that the paper is already quite long, I would seriously consider reducing (or even eliminating) the sections corresponding to the last two points. A general mention can be made about this (in fact, the authors already refer generally to several other experiments which did not prove very useful). Although the definition of the condition measure in this case might be interesting, I think that it doesn't really reflect an ill-posedness situation as the problem can still be solved with no major problem.

The other concern is the following: the authors found not such a strong correlation with $\log C(d)$ (at least compared with $g^m$). However, the theory states that iteration counts is linearly dependent on $\sqrt{\theta}\log C(d)$. So, one might ask whether a better correlation can be detected with respect tp $\sqrt{m}\log C(d)$ or, maybe, $\sqrt{m+n}\log C(d)$, or $(m+n)\log C(d)$. The authors could give some comments on this.

Besides this, I have the following observations:

**pages 1 and 2** In the abstract as well as the introduction, the authors immediately present some of their correlation figures. They use the concept $CORR(\cdot,\cdot)$. Although it is clear what they mean, the concept has not been defined at that point.

**page 2, line 9-12** The authors should state more clearly that the results they claim regarding correlation between condition measures and IPM iterations are from the cited reference.

**page 7** The proof of Proposition 2 could be shortened. I believe that the fact that $x^s$ can be perturbed by a matrix $sI$ for some appropriate $s$, to make it positive semidefinite is a well known fact and doesn't need to be developed.

**page 8** I believe that the dependence on $\epsilon$ of the condition measure $g^m$ should be made explicit, something like $g^m_\epsilon$.

**page 9, line 9 onwards** The stated value of $\epsilon$ depends on $c^T x_k - b^T y_k$, which is the attained duality gap, but which can be bounded by some of the software tolerance parameter. That depends on the stopping criteria, but if we are stopping based on a duality gap of, say, $\bar{\epsilon}$, then we can deduce that we can take $\epsilon = 3/2\bar{\epsilon}$. So, the way we are measuring conditioning with this combined measure might depend on the way we specify the algorithms. I believe that this dependencies should be clarified as for the same $\epsilon$, different instances of a problem will have very different measure $g$ (the case of instances very close to having an unbounded solution set, for instances).

**page 9, line 22** when it said that 32 instances had no primal interior solution, I suppose that that is within the software tolerance.

**page 10, line 4** The expression for $err$ I believe is the definition from SDPT3, but it should be stated clearly.

**page 17, line 9 from below** I suppose the authors wanted to say (SDP) in a more specific form than (CP).

**page 18, line 3** I believe the sentence should say: "must descend at least..."

**pag 23** If the sections on non strict complementarity and degeneracy are eliminated or reformulated, the conclusions will have to be rewritten to preserve consistency.