Referee's Report

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Large-Scale Semidefinite Programming via Saddle Point Mirror-Prox Algorithm

The paper consists of two parts. The first discusses how a positive semidefiniteness requirement on a sparse matrix A of order n with the property that for some $v \in \mathbb{N}^n$, $\sum v_i$ small, all nonzeros $0 \neq a_{ij}$ with $i \leq j$ satisfy $j \leq i + v_i$ (so one such class is matrices with small bandwidth) can be decomposed into the requirement that several small matrices have to be positive semidefinite (which is a completion result for positive semidefinite matrices). The second part shows how this works together (on problems with some favorable structure) with the Mirror-Prox Algorithm of [6] (which, unfortunately, I had not yet a chance to study) so as to obtain solution algorithms with good complexity estimates for ε-solutions. The authors also illustrate the practical efficiency by numerical results on Lovász-ϑ and maxcut instances. Even though computation times look enormous, the instances are indeed large-scale and in particular the Lovász-ϑ instances are typically very difficult to compute in this precision for this size. In fact, I do not know of any other approach so far that could attack them with comparable success — in this statement I assume that the somewhat special structure of the instances is of little influence.

To the best of my knowledge the approach is new and in my eyes very promising and relevant. Even though the paper is written in a very clear and mathematical precise way, I did not find it particularly easy to read, the main difficulty being that lots of notation is introduced on page 4, whose importance and meaning becomes clear only later. I needed to carry through the workings of the definitions on a little example in order to get things arranged correctly in my mind. Maybe it would help to illustrate the definitions by such an example immediately (some examples are give later, but it would be helpful to have one here). I did it for the matrix (I only specify the upper triangle)

$$\begin{bmatrix} * & * & * & * & & & \\ & * & * & & & & \\ & & * & * & * & * & \\ & & & * & * & & \\ & & & & * & * & * \\ & & & & & * & * \\ & & & & & & * \end{bmatrix}$$

but there are certainly even better examples. In several places the notation is also very dense and it would help the reader to digest the formulae by splitting some of them up into a few separate pieces.

After these minor cosmetic corrections the paper is certainly worth publishing.

Details:

- title: something is missing for my language feeling: Large-Scale Semidefinite Programming via a Saddle Point Mirror-Prox Algorithm
- abstract: The last sentence does not sound correct. Maybe write "Implementations and some numerical results for large-scale Lovász ϑ and MAXCUT problems are presented."
- page 1, line 18: sparsity patterns.
- page 1, line 19: (that is, the sparsity patter . . .
- page 2, footnote 1): The full stop is missing.
- page 4, line 18: . . . for which i and j belong to $\underline{J_k}$
- page 4: Please illustrate all definitions on this page by means of a small example.
- page 5, (5): even though it is pretty obvious: please introduce W
- page 6, line 4: needs \normalsize and a full stop.

- page 6, line 9: delete one of the two consecutive $\succeq 0$.
- page 6, line 13: $\dots$, we get an $i_{m-1} \times i_{m-1} \dots$
- Why do you use Card instead of the usual $|\cdot|$? Maybe introduce it in the initial notation paragraph.
- page 7, line 7: $\dots$ if there exists a matrix $\dots$
- page 7, proof of L3.1: Why don't you introduce Δ already in the proof of Prop 3.1 and formulate the Lemma simply as a corollary? In the current form one has to go over the same proof once again to see whether anything changed.
- page 8, line 6: $i = 1, 2\underline{, \dots ,}i_k$
- page 8, (13): The definition of Δ is one of these very dense formulae I alluded to in the text above. It has an unfamiliar appearance to me. Here and later I always needed some time to pick apart the various objects and definitions that are compiled here.
- page 9, line 4: the notation δ^i_j should be explained in the initial notation paragraph.
- page 9, line -4 (the big matrix): Please write zeros into the empty boxes (at first I thought, they should be continued in the same way).
- page 10, Proof of Prop 3.4: For me it would be easier to continue after the first sentence with: Because $\inf_{X \succeq 0} \mathrm{Tr}(XA) \geq 0 \Leftrightarrow A \succeq 0$, this latter requirement is equivalent to $[Z_{ij}]_{i,j \in J_k} \succeq 0$ for $k = 1, \dots, m$.
- page 12, (17): please remind the reader, that $\mathcal{K}$ is defined in (3).
- page 12, line 10: Even if the correct interpretation is pretty obvious, please define ε-solution (there are so many notions of e-solutions around $\dots$).
- page 13, line -10: The k-th diagonal $\dots$
- page 13, line -7: The k-th diagonal $\dots$
- page 13, line -1: Note, I have not checked this since I didn't have [6] at hand (but I have no doubts that it is correct).
- page 14, line -11: Let $v = (v_1, v_{\underline{2}}, \dots$
- page 14, line -7: This sentence is hard to digest and does not quite sound like proper English.
- page 14, (30): as above, I did not check this.
- page 14, (31): This is a bit too easy, since this only refers to the work observed in practice, but it needs a comment that in strict theoretical terms the situation is somewhat more complicated.
- page 17, line 4: Then an ε-solution $\dots$
- page 19, line 5: An opening [is missing.