A One Perturbation Variational Principle and Applications

Abstract

We study a variational principle with a *common* perturbation function φ for *all* proper lower semicontinuous extended real-valued functions f on a metric space X.

- Necessary and sufficient conditions are given for the perturbed $f+\varphi$ to attain its minimum.
- For separable Banach space we may use a perturbation function that is also convex and *Hadamard-like* differentiable.
- We give three applications to differentiability of convex functions on separable and more general Banach spaces.
- We pose various open questions.