The Transshipment Problem

One requirement of the transportation problem is advance knowledge of the method of distributing flow from each source to each destination. This determines the cost. Sometimes, however, the best method of routing the flow is not clear because of the possibility of transshipments.

Continuing our pea shipping example, suppose our company decided to look at using common carriers to ship our goods (common carriers are trucking companies for hire, as opposed to internal trucking divisions). Since no single trucking company serves all of our area, many shipments will need to be transfered to another truck at least once along the way. These transfers can be made at an intermediate cannery or warehouse, or be made at one of three other *junctions*. These are at Butte, Montana; Boise, Idaho; and Cheyenne, Wyoming. The shipping cost per truck is given in the following table:

		Cannery			Junction			Warehouse				Out-put
		1	2	3	1	2	3	1	2	3	4	
	1		146	—	324	286	—	452	505	—	871	75
Cannery	2	146		—	373	212	570	335	407	688	784	125
	3	—	—		658	—	405	—	685	359	673	100
	1	322	371	656		262	398	503	234	329	—	
Junction	2	284	210	—	262		406	305	207	464	558	
	3	—	569	403	398	406		597	253	171	282	
	1	453	336	—	505	307	599		359	706	587	
Ware-house	2	505	407	683	235	208	254	357		362	341	
	3	—	687	357	329	464	171	705	362		457	
	4	868	781	670	—	558	282	587	340	457		
Demand								80	65	70	85	

So, for instance, peas can go from cannery 1 to warehouse 4 directly at cost 871. They can also go from cannery 1, to junction 2, to warehouse 2, and then to warehouse 4 at cost of **286+207+341 = 834**.

The overall problem is to determine how the output from each cannery should be shipped to the warehouses to meet demand and minimize shipping costs.

Note that the above table does not represent a transportation problem. It is a *transshipment problem*. We can, however, represent any transshipment problem as an equivalent transportation problem.

The fundamental idea is to interpret the individual truck trips as being the shipment from a source to a destination. Therefore, there is both a source and a destination for all of the places in our problem. There is a source node for cannery 1 (representing everything leaving cannery 1) and a destination node (representing everything entering the node). This gives rise to the following cost table (ignore supplies and demands for now):

	Cannery			Junction			Warehouse				Out-
	1	2	3	1	2	3	1	2	3	4	put
Cannery 1	0	146	M	324	286	M	452	505	M	871	375
Cannery 2	146	0	M	373	212	570	335	407	688	784	425
Cannery 3	M	M	0	658	M	405	M	685	359	673	400
Junction 1	322	371	656	0	262	398	503	234	329	M	300
Junction 2	284	210	M	262	0	406	305	207	464	558	300
Junction 3	M	569	403	398	406	0	597	253	171	282	300
Warehouse 1	453	336	M	505	307	599	0	359	706	587	300
Warehouse 2	505	407	683	235	208	254	357	0	362	341	300
Warehouse 3	M	687	357	329	464	171	705	362	0	457	300
Warehouse 4	868	781	670	M	558	282	587	340	457	0	300
Demand	300	300	300	300	300	300	380	365	370	385	

Impossible shipments are represented by a large cost **M**. Shipments from a place to itself are assigned cost 0.

The final step is to give every source node a supply and every destination node a demand. Now suppose we knew how much would be shipped through a place. If that amount was 50, then we would increase the supply for the source version of that place by 50 and we would increase the demand for the destination version of that place by 50. This would ensure that 50 units were shipped through that place.

We do not know the amount shipped through each node, but we can place an upper bound on the amount. Since it would not pay to ship something through a place twice, a safe upper bound on the amount shipped through the place is the total supply (300 in this case). This value is added to the supply or demand of each node. So if 50 flow is sent through junction 3, then 250 is sent from the source of junction 3 to the destination of junction 3 (at zero cost). In order to meet supply and demand, 50 units of flow must be sent to the demand node of junction 3, and 50 units of flow are available at the source node and must be sent somewhere else. But the flow through junction 3 can be any value from 0 to 300.

By solving this transportation problem, we get the following optimal solution:

300 units from cannery 1 to cannery 1
75 from cannery 1 to junction 2
300 from cannery 2 to cannery 2
80 from cannery 2 to warehouse 1
45 from cannery 2 to warehouse 2
300 from cannery 3 to cannery 3
70 from cannery 3 to warehouse 3
30 from cannery 3 to warehouse 4
300 from junction 1 to junction 1
225 from junction 2 to junction 2
75 from junction 2 to warehouse 2
300 from junction 3 to junction 3
300 from warehouse 1 to warehouse 1
245 from warehouse 2 to warehouse 2
55 from warehouse 2 to warehouse 4
300 from warehouse 3 to warehouse 3
300 from warehouse 4 to warehouse 4.

Removing the shipments that remain in place leaves:

75 from cannery 1 to junction 2
80 from cannery 2 to warehouse 1
45 from cannery 2 to warehouse 2
70 from cannery 3 to warehouse 3
30 from cannery 3 to warehouse 4
75 from junction 2 to warehouse 2
55 from warehouse 2 to warehouse 4

so only junction 2 and warehouse 3 are used to transfer goods.

General structure. The transshipment problem is concerned with how to allocate and route flow (peas in our example) from supply centers to destinations via intermediate points. In addition to transshiping flow, supply centers generate a surplus that must be distributed and each destination generates a given deficit. Intermediate points (or transshipment nodes) neither generate nor absorb flow. The total supply must equal the total demand, so dummy nodes should be added appropriately. No connection may have a capacity, and all costs should be nonnegative. This defines a transshipment problem.

Transshipment problems can be solved via a transformation to the transportation problem.

Supplementary Exercise 1 *A company must meet the following demands for cash at the beginning of each of the next six months:*

Month	1	2	3	4	5	6
Needs	$200	$100	$50	$80	$160	$140

At the beginning of month 1, the company has $150 in cash and $200 worth of bond 1, $100 worth of bond 2 and $400 worth of bond 3. Of course, the company will have to sell some bonds to meet demands, but a penalty will be charged for any bonds sold before the end of month 6. The penalties for selling $1 worth of each bond are shown in the table below.

		Month of sale					
		1	2	3	4	5	6
	1	$0.07	$0.06	$0.06	$0.04	$0.03	$0.03
Bond	2	$0.17	$0.17	$0.17	$0.11	$0	$0
	3	$0.33	$0.33	$0.33	$0.33	$0.33	$0

(a) *Assuming that all bills must be paid on time, formulate a balanced transportation problem that can be used to minimize the cost of meeting the cash demands for the next six months.*

(b) *Assume that payment of bills can be made after they are due, but a penalty of $0.02 per month is assessed for each dollar of cash demands that is postponed for one month. Assuming all bills must be paid by the end of month 6, develop a transshipment model that can be used to minimize the cost of paying the next six months' bills.*

[Hint: *Transshipment points are needed, in the form* C_t = *cash available at beginning of month* t *after bonds for month* t *have been sold, but before month* t *demand is met. Shipments into* C_t *occur from bond sales and* C_{t-1}. *Shipments out of* C_t *occur to* C_{t+1} *and demands for months* $1, 2, \ldots, t$.]

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