# Assignment Problem

A special case of the transportation problem is the *assignment problem* which occurs when each supply is 1 and each demand is 1. In this case, the integrality implies that every supplier will be assigned one destination and every destination will have one supplier. The costs give the charge for assigning a supplier and destination to each other.

**Example.** A company has three new machines of different types. There are four different plants that could receive the machines. Each plant can only receive one machine, and each machine can only be given to one plant. The expected profit that can be made by each plant if assigned a machine is as follows:

| | Plant | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| Machine 1 | 13 | 16 | 12 | 11 |
| Machine 2 | 15 | 0 | 13 | 20 |
| Machine 3 | 5 | 7 | 10 | 6 |

This is a transportation problem with all supplies and demands equal to 1, so it is an assignment problem.

Note that a balanced problem must have the same number of supplies and demands, so we must add a dummy machine (corresponding to receiving no machine) and assign a zero cost for assigning the dummy machine to a plant.

**Exercise 6** *Albert, Bob, Carl, and David are stranded on a desert island with Elaine, Francine, Gert, and Holly. The "compatibility measures" in the next table indicate the happiness each couple would experience if they spent all their time together. If a couple spends only a partial amount of time together, then the happiness is proportional to the fraction of time spent. So if Albert and Elaine spend half their time together, they earn happiness of 7/2.*

| | $E$ | $F$ | $G$ | $H$ |
|---|---|---|---|---|
| $A$ | 7 | 5 | 8 | 2 |
| $B$ | 7 | 8 | 9 | 4 |
| $C$ | 3 | 5 | 7 | 9 |
| $D$ | 5 | 5 | 6 | 7 |

(a) *Let $x_{ij}$ be the fraction of time that man $i$ spends with woman $j$. Formulate a linear program that maximizes the total happiness of the island (assume that no man spends time with any other man, no woman spends time with any other woman, and no one spends time alone).*

(b) *Explain why exactly four $x_{ij}$ will be 1 and the rest will be 0 in an optimal solution to this linear program. This result is called the* Marriage Theorem *for obvious reasons.*

(c) *Do you think that this theorem will hold if we allow men to spend time with men and women to spend time with women?*

---

*Michael A. Trick*
*Wed Nov 8 14:14:52 EST 1995*