Crashing the Project

It is easy then to use the above linear programming formulation to determine which projects to decrease the time for. Suppose we need to complete the house in 40 days, and we can decrease the excavation by at most 1 day at cost $500/day, foundation by at most 2 days at cost $600 per day, rough walling by at most 2 days at cost $400 per day, and exterior siding by at most 4 days at cost $300 per day decreased. How can we minimize cost so that the house is finished in 40 days?

If we let new variables z_1, z_2, z_3, and z_4 represent the number of days we decrease the time needed for excavation, foundation, walling, and siding, respectively, then we get the L.P.:

$$
\begin{array}{ll}
\text{Minimize} & 500z_1 + 600z_2 + 400z_3 + 300z_4 \\
\text{subject to} & x_2 \geq x_1 + 2 - z_1 \\
& x_3 \geq x_2 + 4 - z_2 \\
& x_4 \geq x_3 + 10 - z_3 \\
& x_5 \geq x_4 + 4 \\
& x_6 \geq x_4 + 6 \\
& x_7 \geq x_4 + 7 \\
& x_7 \geq x_5 + 5 \\
& x_8 \geq x_5 + 0 \\
& x_8 \geq x_6 + 7 - z_4 \\
& x_9 \geq x_7 + 8 \\
& x_{10} \geq x_8 + 9 \\
& x_{11} \geq x_9 + 4 \\
& x_{12} \geq x_9 + 5 \\
& x_{12} \geq x_{11} \\
& x_{13} \geq x_{10} + 2 \\
& x_{13} \geq x_{12} + 6 \\
& x_1 = 0 \\
& x_{13} - x_1 \leq 40 \\
& z_1 \leq 1 \\
& z_2 \leq 2 \\
& z_3 \leq 2 \\
& z_4 \leq 4
\end{array}
$$

PERT

So far we have assumed that reasonably accurate estimates can be made of the time required for each activity in the project network. In reality, ther is frequently some uncertainty about the time an activity can take.

In a PERT network, this uncertainty is summed up in three numbers about each activity: the most likely value for the duration (**m**), a pessimistic value (**b**) and an optimistic value (**a**). PERT then fits a particular type of probability distribution to these values. This distribution (the *beta* distribution) assumes that the range from **a** and **b** encompasses 6 standard deviations (3 on either side of the mean). The mean itself is calculated as

$$\text{Mean} = \frac{a + 4m + b}{6}$$

and the variance:

$$\text{Variance} = \frac{(b-a)^2}{36}.$$

Based on these values, PERT will use an activity network to calculate a mean finishing time along with a variance about that finishing time. There are two critical assumptions: the times for the activities are *independent* of each other, and the critical path identified is always the longest path in the network, no matter how the activity lengths turn out.

With these assumptions, you can solve a PERT network as follows. Find a critical path using the CPM method with the mean activity times on the arcs. This gives the mean finishing time. The variance of the finishing time is simply the sum of the variances of the activities on the critical path. The overall finishing value is assumed to be normally distributed, so quantiles are based on the normal distribution.

PERT allows you to answer such questions as:

- What is the probability the total time is less than 40 days?
- What is the probability the total time is more than 50 days?
- How much should the project budget be increased in order to ensure with 99% probability that the project will finish by December 10?

Unfortunately, the assumptions behind PERT are very stringent. Be careful when using any canned package for it hides the assumptions very well. Used correctly, both PERT and CPM greatly aid in the control and analysis of large projects.

Exercise 7 *Consider the following project.*

Activity	*Duration*	*Predecessors*
A	*6*	—
B	*3*	—
C	*5*	—
D	*4*	*A*
E	*5*	*A*
F	*7*	*B*
G	*4*	*C*
H	*6*	*C*
I	*3*	*D*
J	*4*	*F,G*
K	*3*	*F,G*
L	*2*	*H*
M	*6*	*E,I,J*
N	*5*	*K,L*

Find the early time and late time for each event as well as the total float for each activity. Also identify the critical path.

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