Set Covering

To illustrate this model, consider the following *location* problem: A city is reviewing the location of its fire stations. The city is made up of a number of neighborhoods, as illustrated in Figure 1.

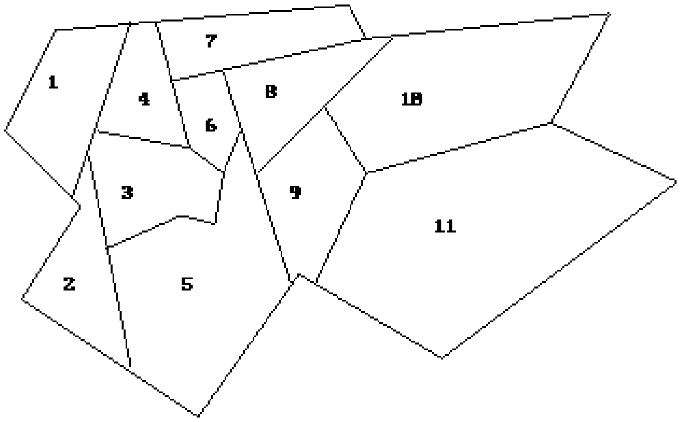

Figure 1: Map of the city

A fire station can be placed in any neighborhood. It is able to handle the fires for both its neighborhood and any adjacent neighborhood (any neighborhood with a non-zero border with its home neighborhood). The objective is to minimize the number of fire stations used.

We can create one variable x_j for each neighborhood **j**. This variable will be 1 if we place a station in the neighborhood, and will be 0 otherwise. This leads to the following formulation

$$
\begin{array}{lr}
\text{Minimize} & x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} \\
\text{subject to} & x_1 + x_2 + x_3 + x_4 \geq 1 \\
& x_1 + x_2 + x_3 + x_5 \geq 1 \\
& x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 1 \\
& x_1 + x_3 + x_4 + x_6 + x_7 \geq 1 \\
& x_2 + x_3 + x_5 + x_6 + x_8 + x_9 \geq 1 \\
& x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \geq 1 \\
& x_4 + x_6 + x_7 + x_8 \geq 1 \\
& x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \geq 1 \\
& x_5 + x_8 + x_9 + x_{10} + x_{11} \geq 1 \\
& x_8 + x_9 + x_{10} + x_{11} \geq 1 \\
& x_9 + x_{10} + x_{11} \geq 1 \\
& x_j \in \{0,1\} \quad j = 1, \ldots, 11
\end{array}
$$

The first constraint states that there must be a station either in neighborhood 1 or in some adjacent neighborhood. The next constraint is for neighborhood 2 and so on. Notice that the constraint coefficient a_{ij} is 1 if neighborhood **i** is adjacent to neighborhood **j** or if **i=j** and 0 otherwise. The **j**th column of the constraint matrix represents the set of neighborhoods that can be served by a fire station in neighborhood **j**. We are asked to find a set of such subsets **j** that *covers* the set of all neighborhoods in the sense that every neighborhood appears in the service subset associated with *at least* one fire station.

One optimal solution to this is $x_3 = x_8 = x_9 = 1$ and the rest equal to 0.

This is an example of the *set covering problem*. The set covering problem is characterized by having binary variables, $\geq$ constraints each with a right hand side of 1, and having simply sums of variables as constraints. In general, the objective function can have any coefficients, though here it is of a particularly simple form.

- Set Packing and Partitioning

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