Capital Budgeting

Suppose we wish to invest $14,000. We have identified four investment opportunities. Investment 1 requires an investment of $5,000 and has a present value (a time-discounted value) of $8,000; investment 2 requires $7,000 and has a value of $11,000; investment 3 requires $4,000 and has a value of $6,000; and investment 4 requires $3,000 and has a value of $4,000. Into which investments should we place our money so as to maximize our total present value?

As in linear programming, our first step is to decide on our variables. This can be much more difficult in integer programming because there are very clever ways to use integrality restrictions. In this case, we will use a 0-1 variable x_j for each investment. If x_j is 1 then we will make investment **j**. If it is 0, we will not make the investment. This leads to the 0-1 programming problem:

$$\begin{array}{rl} \text{Maximize} & 8x_1 + 11x_2 + 6x_3 + 4x_4 \\ \text{subject to} & 5x_1 + 7x_2 + 4x_3 + 3x_4 \leq 14 \\ & x_j \in \{0,1\}\ j = 1, \ldots 4. \end{array}$$

Now, a straightforward ''bang for buck'' suggests that investment 1 is the best choice. In fact, ignoring integrality constraints, the optimal linear programming solution is $x_1 = 1, x_2 = 1, x_3 = 0.5, x_4 = 0$ for a value of $22,000. Unfortunately, this solution is not integral. Rounding x_3 down to 0 gives a feasible solution with a value of $19,000. There is a better integer solution, however, of $x_1 = 0$, $x_2 = x_3 = x_4 = 1$ for a value of $21,000. This example shows that rounding does not necessarily give an optimal value.

There are a number of additional constraints we might want to add. For instance, consider the following constraints:

1. We can only make two investments.
2. If investment 2 is made, investment 4 must also be made.
3. If investment 1 is made, investment 3 cannot be made.

All of these, and many more *logical restrictions*, can be enforced using 0-1 variables. In these cases, the constraints are:

1. $x_1 + x_2 + x_3 + x_4 \leq 2$
2. $x_2 - x_4 \leq 0$

3. $x_1 + x_3 \leq 1$.

Exercise 1 (Optional) *As the leader of an oil exploration drilling venture, you must determine the best selection of 5 out of 10 possible sites. Label the sites* $s_1, s_2, \ldots, s_{10}$ *and the expected profits associated with each as* $p_1, p_2, \ldots, p_{10}$.

(i) If site s_2 *is explored, then site* s_3 *must also be explored. Furthermore, regional development restrictions are such that*

(ii) Exploring sites s_1 and s_7 *will prevent you from exploring site* s_8.

(iii) Exploring sites s_3 or s_4 *will prevent you from exploring site* s_5.

Formulate an integer program to determine the best exploration scheme.

- Multiperiod Capital Budgeting

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