Solutions to Some Optional Problems

Exercise 1: The formulation is:

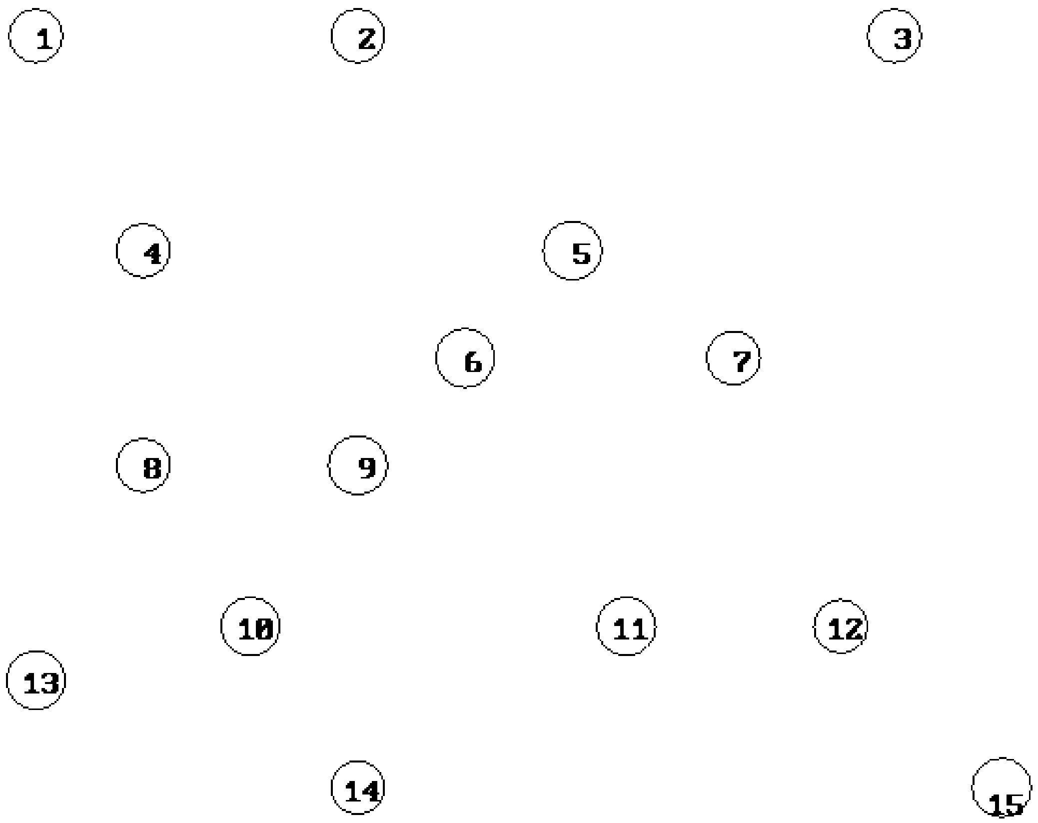


Figure 1: Map of the 15 cities

Exercise 2: Let be a **0,1** variable which

$$\begin{array}{lll} \text{Maximize} & \sum_{j=1}^{10} p_j x_j & \\ \text{subject to} & \sum_{j=1}^{10} x_j & = \; 5 \\ & x_2 - x_3 & \leq \; 0 \\ & x_1 + x_7 + x_8 & \leq \; 2 \\ & x_3 + x_5 & \leq \; 1 \\ & x_4 + x_5 & \leq \; 1 \\ & x_j & \in \; \{0,1\} \;\; j = 1, \ldots, 10. \end{array}$$

equals 1 if route **j** is used, 0 otherwise. Then the vehicle routing problem can be formulated as a set covering problem:

$$x_j$$

Exercise 3: Let starting

$$\begin{array}{lll} \text{Minimize} & 10x_1 + 12x_2 + 12x_3 + 13x_4 + 11x_5 & \\ & \quad + 9x_6 + 7x_7 + 8x_8 + 8x_9 & \\ \text{subject to} & x_1 + x_2 + x_3 + x_7 & \geq \; 1 \\ & x_3 + x_4 + x_5 + x_8 & \geq \; 1 \\ & x_1 + x_4 + x_6 + x_8 & \geq \; 1 \\ & x_1 + x_2 + x_5 + x_6 + x_9 & \geq \; 1 \\ & x_2 + x_3 + x_4 + x_5 + x_6 + x_9 & \geq \; 1 \\ & x_j & = \; 0 \text{ or } 1 \text{ for all } j. \end{array}$$

time of operation **j**.

$$x_j =$$

Then the formulation is:

$$y_{ij} = \begin{cases} 0 \text{ if operation } j \text{ is performed before } i \\ 1 \text{ if operation } i \text{ is performed before } j. \end{cases}$$

The first set of constraints are the precedence constraints and the last two sets are the noninterference constraints.

Exercise 4: First we solve the problem as a linear program with LINDO. The solution is

$$
\begin{array}{lll}
\text{Minimize} & x_8 & \\
\text{subject to} & x_i + p_i \le x_j & \text{for } (i,j) = (1,4),(2,4),(3,6), \\
& & (4,7),(5,7),(5,8),(6,8) \\
& x_7 + p_7 \le d_1 & \\
& \left.\begin{array}{r} My_{ij} + x_i - x_j \ge p_j \\ M(1-y_{ij}) + x_j - x_i \ge p_i \end{array}\right\} & \text{for } (i,j) = (1,2),(1,3),(1,5), \\
& & (1,6),(1,8),(2,3),(2,5),(2,6), \\
& & (2,8),(3,4),(3,5),(3,7),(4,5), \\
& & (4,6),(4,8),(5,6),(6,7),(7,8) \\
& x_j = 0 \text{ or } 1 & \text{for } j = 1,\ldots,8.
\end{array}
$$, $x_1 = 3.33333$.

Rounding it yields $x_2 = x_3 = 0, x_1 = 3$ which fails to satisfy the constraint $x_2 = x_3 = 0$.

In fact, the only feasible integer solution is

$$6x_1 + 20x_2 + 4x_3 = 20$$

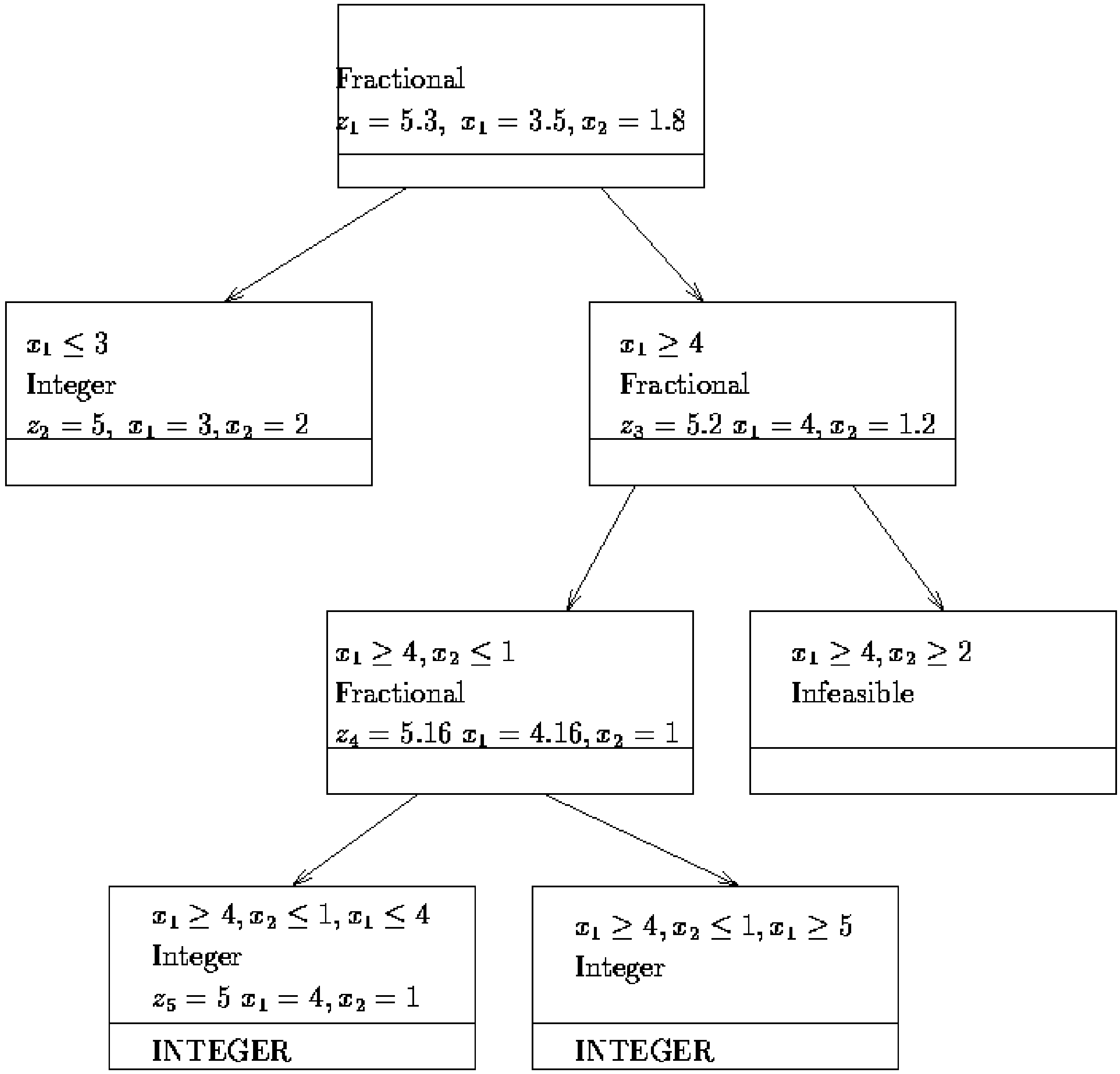
and it cannot be obtained by simple rounding.

Exercise 5: The enumeration tree is given in Figure 9.

$x_1 = 2, x_2 = 0, x_3 = 2,$

Figure 9: Enumeration tree

Three optimum integer solutions were found, namely



each with value **z=5**.

Exercise 6

From the enumeration tree of Exercise 5, we see that only one branching is necessary since both subproblems $\begin{cases} x_1 = 3 \\ x_2 = 2 \end{cases}$ $\begin{cases} x_1 = 4 \\ x_2 = 1 \end{cases}$ $\begin{cases} x_1 = 5 \\ x_2 = 0 \end{cases}$ and z_2 are inactive by reason of integrality. The better solution is z_3 with value $x_1 = 4, x_2 = 1.2$.

Exercise 7

```
MAX 4 X1 + 7 X2 + 6 X3 + 5 X4 + 4 X5
SUBJECT TO
   2) 5 X1 + 8 X2 + 3 X3 + 2 X4 + 7 X5 <= 112
   3) X1 + 8 X2 + 6 X3 + 5 X4 + 4 X5 <= 109
END
GIN 5

ENUMERATION COMPLETE. BRANCHES= 6 PIVOTS= 20

OBJECTIVE FUNCTION VALUE  151.000

VARIABLE      VALUE      REDUCED COST
      X1      14.000           -4.000
      X2        .000           -7.000
      X3        .000           -6.000
      X4      19.000           -5.000
      X5        .000           -4.000
```

Exercise 8

Since $z_3 = 5.2$ does not satisfy the knapsack constraint $x_1 = x_2 = x_3 = 1$, it follows that $5x_1 + 7x_2 + 4x_3 + x_4 \leq 14$ must hold for all solutions of the knapsack problem.

Similarly for $x_1 + x_2 + x_3 \leq 2$.

```
MAX 8 X1 + 11 X2 + 6 X3 + 4 X4
SUBJECT TO
   2) 5 X1 + 7 X2 + 4 X3 + 3 X4  <= 14
   3) X1 +  X2 +  X3 <= 2
   3) X1 +  X2 +  X4 <= 2
END
SUB X1 1
SUB X2 1
SUB X3 1
SUB X4 1

OBJECTIVE FUNCTION VALUE  21.000

VARIABLE      VALUE      REDUCED COST
      X1        .000            0.000
      X2       1.000           -3.000
      X3       1.000           -2.000
      X4       1.000            0.000
```

Exercise 9.

(a) Nearest Neighbor starting from city 1 yields the tour

1 -- 4 -- 8 -- 9 -- 10 -- 13 -- 14 -- 11 -- 12 -- 15 -- 7 -- 5 -- 6 -- 2 -- 3 -- 1

of length 89.

Applying 2-opt, we get the following improvements: replace 1--3 and 2--6 by 1--2 and 3--6 for a gain of

6. The new tour is

1 -- 2 -- 3 -- 6 -- 5 -- 7 -- 15 -- 12 -- 11 -- 14 -- 13 -- 10 -- 9 -- 8 -- 4 -- 1

Replace 3--6 and 5--7 by 3--5 and 6--7 for a gain of 2. The new tour

1 -- 2 -- 3 -- 5 -- 6 -- 7 -- 15 -- 12 -- 11 -- 14 -- 13 -- 10 -- 9 -- 8 -- 4 -- 1

has lenth 81.

(b) We start Nearest Insertion with the tour 5 -- 6 -- 5 of length 6.

Then we insert city 7 at a cost of 4+5-3=6 (note that there is a tie. We could have inserted city 9 instead). The new tour is 5 -- 6 -- 7 -- 5. Then the nodes are inserted in the following order: 9 (between 6 and 7), 11 (between 7 and 9), 12 (between 7 and 11), 10 (between 9 and 11), 14 (between 10 and 11), 8 (between 9 and 10), 13 (between 8 and 10), 4 (between 8 and 9), 2 (between 4 and 9), 1 (between 2 and 4), 15 (between 11 and 12) and finally 3 (between 5 and 7). The resulting tour is

1 -- 2 -- 9 -- 6 -- 5 -- 3 -- 7 -- 12 -- 15 -- 11 -- 14 -- 10 -- 13 -- 8 -- 4 -- 1

and has length 79. There is no improvement with 2-opt.

(c) We start Furthest Insertion with the tour 1 -- 15 -- 1 of length 46. Then city 3 is inserted followed by city 13. The new tour is 1 -- 3 -- 15 -- 13 with length 60. The remaining nodes are inserted in the folowing order: 6 (between 13 and 15), 14 (between 6 and 13), 5 (between 6 and 15), 11 (between 5 and 15), 7 (between 5 and 11), 10 (between 13 and 14), 9 (between 6 and 14), 12 (between 11 and 15), 12 (between 11 and 15), 4 (between 1 and 13), 8 (between 4 and 13) and finally 2 (between 1 and 3). The resulting tour

1 -- 2 -- 3 -- 15 -- 12 -- 11 -- 7 -- 5 -- 6 -- 9 -- 14 -- 10 -- 13 -- 8 -- 4 -- 1

has length 81. There is no improvement with 2-opt.

(d) We take as the central point on the map the midpoint between 5 and 10. The resulting tour obtained by applying the sweep heuristic is

7 -- 3 -- 5 -- 6 -- 2 -- 1 -- 4 -- 8 -- 9 -- 13 -- 10 -- 14 -- 11 -- 15 -- 12 -- 7

and it has length 80.

Applying the 2-opt heuristic, we replace 9--13 and 10--14 by 9--10 and 13--14, thus reducing the length of the tour by 1. The new tour

1 -- 2 -- 6 -- 5 -- 3 -- 7 -- 12 -- 15 -- 11 -- 14 -- 13 -- 10 -- 9 -- 8 -- 4 -- 1

has length 79.

Michael A. Trick
Thu Oct 26 12:05:11 EDT 1995