Assignment 5, CO666

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1. Suppose that $A, B \in \mathcal{S}^n$ are two given symmetric $n \times n$ matrices and there exists $x \in \mathcal{R}^n$ such that $x^t Ax > 0$. Prove that the following two statments are equivalent:

   (a)

   $$x^t Ax \geq 0 \text{ implies } x^t Bx \geq 0$$

   (b) There exists $\lambda \geq 0, \Lambda \succeq 0$ (positive semidefinite) such that

   $$B = \lambda A + \Lambda.$$

2. Suppose that $A, B, C \in \mathcal{S}^n$ are three given symmetric $n \times n$ matrices and there exists $x \in \mathcal{R}^n$ such that both $x^t Ax > 0$ and $x^t Bx > 0$. Show by example that the statement

   $$x^t Ax \geq 0, x^t Bx \geq 0 \text{ implies } x^t Cx \geq 0$$

   does **not** imply the existence of $\lambda, \mu \geq 0$, such that $C \succeq \lambda A + \mu B$.

3. Bertsekas Page 436 #5.1.6