# C&O 666
# Assignment 2

Due on Tuesday, Mar. 14, Instructor H. Wolkowicz

1. Suppose that $C$ is a convex set in $\Re^n$ and the point $y \notin \bar{C}$, the closure of $C$. Prove that there exists $\phi$ such that
$$\phi^t y < \phi^t x, \quad \forall x \in C.$$
(Hint: First prove that there is a point $\bar{x} \in \bar{C}$ which is closest to $y$.)

2. Suppose that $K_1, K_2$ are convex sets in $\Re^n$ and the interior of $K_1$ is not empty, $\operatorname{int}(K_1) \neq \emptyset$. In addition, suppose that
$$K_2 \cap \operatorname{int}(K_1) = \emptyset.$$
Prove that there exists a hyperplane separating $K_1, K_2$.

3. Suppose that $S \subset \Re^n$. Prove that $S^{++} = S$ if and only if $S$ is a closed convex cone. ($\cdot^+$ denotes polar)

4. Consider the trust region subproblem (TRS)
$$\min x^t A x - 2b^t x \text{ subject to } x^t x - \delta^2 \leq 0,$$
where $A = A^t$. Using the min-max and max-min approach, derive a dual program to (TRS) which is a simple constrained concave maximization program. Then derive the dual to this dual and relate it to the original (TRS) problem.