C&O 370
Deterministic OR Models
Winter 1996

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Lectures: (01) 8:30 MWF, MC 4063
(02) 10-11:30 TR, MC 4060

Texts: Applied Mathematical Programming, Bradley, Hax and Magnanti
GAMS, A User's Guide, Brooke, Kendrick and Merraus.

References: Linear Programming, Chvátal
Intro. to Math. Programming, Hillier & Lieberman
Home Page, C&O 370,
ftp://orion.uwaterloo.ca/pub/henry/teaching/w96/370.w96/readme.ht ml

Term Work: Term work will consist of homework problems, case studies and a research project.
See attached schedule of assignments, due dates and instructions.

Midterm Exam: A 2-hour exam, 7-9, on Friday, March 1, in MC 4020/MC 4021.

Final Exam: A 3-hour exam, scheduled by the registrar.

Marking Scheme:

Homework	10%
Case Studies	20%
Research Project	20%
Midterm Exam	20%
Final Exam	30%

C&O 370
HOMEWORK

There will be 4 homework assignments given throughout the term. Each will consist of problems and specified readings from the text, references and resource materials listed on the C&O 370 homepage. The problems are to be done individually and handed in for grading.

Some problems will be computational exercises giving practice with implementing general algorithms, using available software, and others will be modelling exercises giving practice with extracting precise mathematical formulations from applied problems.

Format for Computational Exercises

- Problem Statement
- Solution Summary
- Outline of Solution Method

Format for Modelling Exercises

- Problem Statement
- Solution Summary (if appropriate)
- Model Development (see section on Case Studies for details)
- Brief Model Critique

The T.A.'s will mark a subset of the problems and will hold office hours to help resolve any difficulties you may have.

The 10% contribution of homework to your course grade belies its importance, as both exams will be based upon homework readings and problems.

The following pages give the homework assignments and their due dates.

C&O 370

Homework #1 (Linear Programming)

Due: Monday (Tuesday), January 22 (23)

Reading

- B/H/M, pp. 48-181 (LP/Sensitivity/Duality)
- B/H/M, pp. 690-703 (Revised Simplex Algorithm)
- Chvátal, pp. 97-105 (Revised Simplex Algorithm)
- □ , FAQ on linear programming.

Problems

- 1.1 B/H/M, p. 33, #5
- 1.2 B/H/M, pp. 46-47, #28(b) & (c) (see Chvátal, pp. 213-227)
- 1.3 B/H/M, pp. 136-137, #14
- 1.4 B/H/M, pp. 193-194, #6

Homework #2 (Network Programming)

Due: Monday (Tuesday), February 12 (13)

Reading

- B/H/M, pp. 310-326 (Network Models)
- Chvátal, pp. 291-299 (Transshipment Problem)
- Chvátal, pp. 367-378 (Maximum Flow Problem)
- □ , shortest path, maximum flow problems
- □ , information and software on network models
- □ , linear network optimization at NEOS

Problems

- 2.1 B/H/M, p. 354, #8
- 2.2 B/H/M, p. 356, #13
- 2.3 B/H/M, p. 359, #19
- 2.4 Chvátal, p. 388, #22.9

Homework #3 (Mixed Integer Programming)

Due: Monday (Tuesday), March 4 (5)

Reading

- B/H/M, pp. 366-385 (MIP Models)
- B/H/M, pp. 402-407 (Cutting Planes)
- Chvátal, pp.195-211 (Cutting Stock Problem)
- □ , systems analysis laboratory
- □ , optimization technology center

Problems

- 3.1 B/H/M, pp. 408-409, #6
- 3.2 B/H/M, p. 496, #16
- 3.3 B/H/M, p. 414, #18
- 3.4 Chvátal, pp. 211-212, #13.3

Homework #4 (Nonlinear Programming)

Due: Monday (Tuesday), March 25 (26)

Reading

- B/H/M, pp. 561-608
- □ , FAQ on nonlinear programming
- □ , Introduction to Convex Optimization
- □ , Newton-Raphson Method

Problems

- 4.1 B/H/M, p. 608, #2
- 4.2 B/H/M, p. 611, #7
- 4.3 B/H/M, pp. 622-623, #28
- 4.4 B/H/M, pp. 623-624, #30

C&O 370
CASE STUDIES

Case Studies are definitive problems requiring a greater degree of analysis and thoroughness of presentation than do homework problems. Generally, computer software such as GAMS will be a required component of such problems. Case Studies should be formal reports, as if done for a "customer". They should be concise, clearly written and self-contained.

Format for Case Studies

- Problem Statement
- Solution Summary
- Model
  - Assumptions
  - Index Sets and Variables
  - Data
  - Program (objective function & constraints, clearly expressed and annotated)
- Assessment of Solution Validity
- Critique of Model Choice and Assumptions
- Appendices (e.g. GAMS Program & Output)

Each Case Study should be done in groups of 2 or 3 students. These groups may be comprised of students from either of the two sections of the course.

On the given due dates each group should submit one paper only, with each group member's name, I.D. and course section number on the title page.

Each case study will be graded and returned to you. The grading will be based upon both technical accuracy and clarity of presentation. It is imperative that you stretch your communication skills.

The following pages give the Case Studies and their due dates.

C&O 370

Case Study #1 (Due: Wednesday (Thursday), January 24 (25))

An *Exgon* oil refinery produces four types of raw gasoline: alkylate, catalytic-cracked, straight-run and isopentane. Two important characteristics of each gasoline are performance number PN (measuring antiknock properties) and vapor pressure RVP (measuring volatility). These characteristics, together with the production limits in barrels per day, are as follows:

	PN	RVP	Production Limit
Alkylate	107	5	3,814
Catalytic-Cracked	93	8	2,666
Straight-Run	87	4	4,016
Isopentane	108	21	1,300

These gasolines can be sold either raw, at \$4.83 per barrel, or blended into aviation gasolines (Avgas A and/or Avgas B). Quality standards impose certain requirements on the aviation gasolines; these requirements, together with the selling prices, are as follows:

	PN	RVP	Price per Barrel
Avgas A	at least 100	at most 7	\$6.45
Avgas B	at least 91	at most 7	\$5.91

The PN and RVP of each mixture are simply weighted averages of the PNs and RVPs of that mixture's constituents. For example, if 1,000 barrels of Avgas A was a blend of 600,100,200 and 100 barrels of Alkylate, Catalytic-Cracked, Straight-Run and Isopentane, respectively, then:

$$\text{PN (AvgasA)} = \frac{(600 \times 107) + (100 \times 93) + (200 \times 87) + (100 \times 108)}{1,000} = 101.7$$

$$\text{RVP (AvgasA)} = \frac{(600 \times 5) + (100 \times 8) + (200 \times 4) + (100 \times 21)}{1,000} = 6.7$$

(a) Develop and solve a GAMS-like model by which *Exgon* can realize a maximum daily sales revenue from this refinery.

(b) Write down the dual of your model from (a), and state the units of the dual variables corresponding to the production-limit constraints.

(c) Another oil company has offered to sell additional quantities of the four raw gasoline types to *Exgon* at specified prices. Analyze how *Exgon* should assess their offer. Illustrate your analysis numerically by choosing the specified prices and *Exgon's* profit margins.

C&O 370

Case Study #2 (Due: Wednesday (Thursday), February 14 (15))

A university has three professors who each teach four courses per year. Each year four sections each of marketing, finance, and production must be offered. At least one section of each of these three courses must be offered in each of the Fall and Spring terms. Each professor's preferences for courses and teaching terms are given in Table 1 below.

	Professor 1	Professor 2	Professor 3
Fall preference	3	5	4
Spring preference	4	3	4
Marketing preference	6	4	5
Finance preference	5	6	4
Production preference	4	5	6

Table 1. Preference Data

The total satisfaction a professor earns teaching a course is the sum of the term preference number and course preference number.

Develop and solve a GAMS-like model by which the dean can assign professors to courses so that the total satisfaction of the three professors is a maximum. Include a carefully drawn and labelled network upon which your model is based.

C&O 370

Case Study #3 (Due: Wednesday (Thursday), March 6 (7))

In women's gymnastics competition there are four events: vault, uneven bars, balance beam and floor exercise. Each team can enter up to six gymnasts in each of these events. Only the top five scores from the six entrants in each event are included in the team score. A major constraint in selecting the six entrants for each event is that there must be at least four "all-arounders" who must compete in all four events.

Develop and solve a model for assisting the coach in selecting the entrants for each event so as to maximize the expected total team score. Test your model using the following data:

	Events			
Gymnast	Vault	Bars	Beam	Floor
1	9.80	9.10	9.15	8.90
2	9.00	8.90	9.10	8.50
3	8.50	8.30	9.20	9.00
4	7.00	9.00	8.30	8.65
5	9.40	8.30	9.00	8.50
6	8.90	8.80	7.90	9.00
7	8.50	8.90	8.80	8.10
8	9.10	9.20	8.50	8.20
9	7.90	7.30	8.00	8.80
10	8.90	8.80	9.10	8.70

Table: Scores (0 to 10 in each event) Expected by the Coach for each Gymnast in each Event.

Additionally, the coach would like to know the second best and third best solutions to this problem.

Case Study #4 (Due: Wednesday (Thursday), March 27 (28))

Do problem #24, pp. 619-620 of B/H/M.

C&O 370
RESEARCH PROJECTS

General: In groups of three or less, you are to solve one of the attached problems. You may use any resources as aids, provided you reference them carefully. You may consult with instructors/tutors concerning your planned approach and to obtain limited technical help. Projects should be clearly written self-contained reports with the narrative parts typed in English.

Guidelines: The project problems tend to be open-ended and are unlikely to have unique solutions. It is a good idea to solve completely a restricted, or core, version of your problem by making appropriate simplifying assumptions and then to do what you can with each of a progression of harder versions, or extensions, obtained by relaxing the assumptions.
Your project will be assessed primarily in terms of how well you communicate your knowledge of both the problem and your model. Accordingly, make your analysis as thorough as possible, and make certain that your model is an appropriate one.

Format: The format should be similar to that of the case studies, with the added components of a detailed model critique and analyses of core problem extensions. The report structure should include:

title page/table of contents/abstract/summary of results/main body/references/appendices.

The summary of results should offer motivation for the reader to want to read the rest of the report. It begins on a separate page (following the abstract), which is page number 1.
The main body should contain your model(s), clearly developed and annotated, your core problem analysis, your extensions and avenues for further study. Divide the main body into subsections as appropriate.
References (not a bibliography) are associated with the citations in the text. Include books, papers, software and personal consultations.
Appendices should be used for such things as extended background material, programmed versions of your model(s) and output and any kind of lengthy detail that would interfere with the flow of text.
Include, as a separate enclosure, a reproduction of presentation materials which you would use for a 15 minute oral presentation of your work.
Well in advance of the project due date arrange a meeting with your professor to discuss your overall study plan, your choice of model(s) and the resources which you anticipate using.

Project Due Date: Monday (Tuesday), March 18 (19), in class.
Presentation Dates: Monday, March 25 through Friday, March 29.

C&O 370
RESEARCH PROJECTS

1. Aluacha Balaclava College
   (see pages 16 and 17)

2. Snow Removal
   After a snowfall, three snowplows are dispatched to the Old Beechwood area of Waterloo. All streets in that area must be plowed twice, one in each direction, in order to remove all of the snow.

   Develp and solve a model for determining how the three plows should be used in order to finish all plowing in the minimum amount of time.

3. Oil Lease Problem
   An oil company has shown an interest in an oil field containing several potential oil wells. The dollar value of each well is assumed known based upon estimated production. A plot of land of known dimensions containing each well must be leased by the company if the well is selected. The plots of land associated with various sets of wells may overlap. There is a major road running parallel to the southern boundary of the oil field, and a side road must be built joining any selected well to this major road.

   The net value of any set of selected wells is the sum of the values of the wells minus the land lease and side road construction costs.

   Develop a model by which the oil company can select a set of wells of maximum net value. Illustrate your model numerically using parameter values of your choice.

4. <u>File Transfers</u>

In your company, information is shared among departments on a daily basis. This information includes the previous day's sales statistics and current production guidance. It is important to get this information out as quicly as possible.

Suppose a communications network is to be used to transfer blocks of data (files) from one computer to another. As an example, consider the following graph model:

Figure 1. Example of a file transfer network.

Vertices $V_1, V_2, \ldots, V_m$ represents computers, edges $e_1, e_2, \ldots, e_n$ represents files to be transferred (between computers represented by edge endpoints). $T(e_x)$ is the time it takes to transfer file e_x and $C(V_y)$ is the capacity of the computer represnted by V_y to simultaneously transfer files. A file transfer involves the engagement of both computers for the entire time it takes to transfer the file. $C(V_y) = 1$;means that computer V_y can only be involved in one transfer at a time.

We are interested in scheduling the transfers in an optimal way to minimize the total time it takes to complete them all. This minimum total time is called the *makespan.*

Your corporation has 28 departments. Each department has a computer, each of which is represented by a vertex in Figure 2. Each day, 27 files must be transferred, represented by the edges in Figure 2. For this network, $T(e_x) = 1$ and $C(V_y) = 1$ for all x and y. Find an optimal schedule and the makespan for the given network.

Figure 2. Network for your corporation.

Find an optimal schedule and the makespan for the same network but with the following transfer times:

$x =$	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$T(e_x)$	3.0	4.1	4.0	7.0	1.0	8.0	3.2	2.4	5.0	8.0	1.0	4.4	9.0	3.2
$x =$	15	16	17	18	19	20	21	22	23	24	25	26	27	
$T(e_x)$	2.1	8.0	3.6	4.5	7.0	7.0	9.0	4.2	4.4	5.0	7.0	9.0	1.2	

5. Loading Flat Cars
Railroad flat cars are to be loaded with seven types of packing crates. The crates have the same width and height but varying thickness (t, in cm) and weight (w, in kg). The table gives, for each crate, the thickness, weight and number available. Each car has 10.2 meters of length available for packing the crates (like slices of toast) and can carry up to 40 metric tons. There is a special constraint on the total number of C_5, C_6, C_7

crates because of a subsequent local trucking restriction: the total space (thickness) occupied by these crates must not exceed 302.7 cm per flatcar. Additionally, at most 4 crate types may be loaded on any flatcar.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	
t	48.7	52.0	61.3	72.0	48.7	52.0	64.0	cm
w	2000	3000	1000	500	4000	2000	1000	kg
	16	14	18	12	12	8	16	

6. Operational Analysis
   Select a local business, and determine in what ways it might benefit from some sort of operational analysis. For example, one might select a business such as **Domino's Pizza** and determine in what ways its delivery policy might be made more profitable. Alternatively, one might select a manufacturing business such as **Brick Brewery** and determine in what ways its production-inventory schedule might be improved.