C&O 350 Assignment 2, Due Tuesday, January 30, 1996

1. Solve the following problem by the simplex method, identifying the basis matrix A_B and the basis inverse A_B^{-1} at each iteration. Is the optimal solution unique? Explain. You may use LPmovie to check your calculations, but do not submit the output.

$$\begin{aligned} \text{maximize } z = -x_1 + 2x_2 + 5x_3 + x_4 & \\ \text{subject to} & \\ x_1 + 2x_2 + 2x_3 - x_4 & \leq 20 \\ -x_1 + x_2 + x_3 + x_4 & \leq 12 \\ 2x_1 + x_2 + x_3 - x_4 & \leq 30 \\ x_j \geq 0, \ \forall j & \end{aligned}$$

2. For the system of equations

$$\begin{array}{cccccccc} x_1 & +x_2 & +x_3 & +x_4 & & = & 4 \\ x_1 & -x_2 & & x_4 & +x_5 & = & 1 \\ x_1 & +x_2 & +x_3 & & +x_5 & = & 4 \end{array}$$

which of the following is a basic solution? Justify your answers.

(a) $(0,1,2,1,1)^T$;

(b) $(2,1,1,0,0)^T$;

(c) $(0,-1,5,0,0)^T$;

(d) $(0,0,\frac{7}{2},\frac{1}{2},\frac{1}{2}))^T$.

3. Give an example to show that the variable that becomes basic on one iteration of the simplex method, can become nonbasic on the next iteration.

4. Give a formula for the reduced cost $\bar{c}_j$ of a nonbasic variable in a simplex tableau (or dictionary) in terms of the coefficients in the previous tableau. Use this formula to prove that the variable that becomes nonbasic on an iteration of the simplex method cannot become basic on the next iteration.

5. (a) Formulate a Phase 1 (auxiliary) problem for finding a nonnegative solution of the system of equations in problem 2. (Note that the method used in Chapter 3 of the text will not work for this. Use the method given in class.)

   (b) Use LPmovie, with the largest coefficient rule for choosing the entering variable, to solve the problem in (a), and submit the output.

   (c) Using the solution to (b) find a feasible tableau or dictionary for the problem maximize $z = x_1 + x_2 + 2x_3 + x_5$ subject to $x_1, \ldots, x_5 \geq 0$ and the equations of problem 2.

6. Let $\bar{x}$ be an element of a convex set S. Show that $\bar{x}$ is an extreme point of S if and only if the set $S - \{\bar{x}\}$ is a convex set.