C&O 350 Assignment 3, Due Tuesday Feb. 13, 1996

1. Solve the following problems by the two phase simplex method. You may use lpmovie to do the calculations, but do not submit output from it.

   (a)

$$
\begin{array}{ll}
\text{minimize} & z = 3x_1 - 4x_2 - x_3 - 2x_4 - 3x_5 \\
\text{subject to} & \\
& \begin{array}{rcr}
x_1 + x_2 + x_3 - x_4 + 2x_5 & \leq & 12 \\
x_1 - 2x_2 - x_3 - x_4 - x_5 & \geq & -30 \\
x_j \geq 0, \ \forall j & &
\end{array}
\end{array}
$$

   (b)

$$
\begin{array}{ll}
\text{maximize} & z = 3x_1 - x_2 - 3x_3 + x_4 \\
\text{subject to} & \\
& \begin{array}{rcr}
x_1 + 2x_2 - x_3 + x_4 & = & 0 \\
2x_1 - 2x_2 + 3x_3 + 3x_4 & = & 9 \\
x_1 - x_2 + 2x_3 - x_4 & = & 6 \\
x_j \geq 0, \ \forall j & &
\end{array}
\end{array}
$$

2. Consider the following problem.

$$
\begin{array}{ll}
\text{maximize} & z = 2x_1 + x_2 \\
\text{subject to} & \\
& \begin{array}{rcr}
x_1 - x_2 & \geq & 8 \\
2x_1 + 3x_2 & \leq & 24 \\
2x_1 + x_2 & \leq & 12 \\
x_j \geq 0, \ \forall j & &
\end{array}
\end{array}
$$

   (a) Verify graphically that this problem is infeasible.

   (b) Use phase I of the two-phase method to show that the problem is infeasible.

3. (a) Suppose that we have a feasible dictionary or tableau for an l.p. problem, corresponding to basis B and basic feasible solution x^*. Show that if $\bar{c}_j < 0$ for every $j \in N$, then x^* is the unique optimal solution of the l.p. problem.

(b) Give an example to show that the converse of the statement in part (a) is not true, that is, that it is possible for x^* to be the unique optimal solution even though some $\bar{c}_j \geq 0$.

4. Arrange the following vectors in lexicographical order, from smallest to largest. $(0,0,1)$, $(0,-1,9)$, $(1,-5,-6)$, $(0,0,0)$, $(0,-1,8)$, $(-1,2,8)$

5. Choose the entering and leaving variables in this example, assuming that we are using:

(a) The smallest subscript rules. (In this case the ε columns should be ignored.)

(b) The largest coefficient rule to choose the entering variable and the lexicographical rule for choosing the leaving variable (using the ε columns).

(c) The smallest subscript rule is used for choosing the entering variable and the lexicographic rule for choosing the leaving variable.

$$\begin{array}{cccccccccccc}
 & & 2x_2 & +x_3 & & -x_5 & +x_6 & = 4 & +2\varepsilon_1 & -2\varepsilon_2 & & \\
 & & x_2 & -x_3 & +x_4 & +2x_5 & & = 2 & +2\varepsilon_1 & & +\varepsilon_2 & +3\varepsilon_3 \\
 & x_1 & +2x_2 & +2x_3 & & +3x_5 & & = 6 & +3\varepsilon_1 & & & \\
z & & -2x_2 & +2x_3 & & -3x_5 & & = 5 & & & &
\end{array}$$

6. Suppose that, when solving a problem with the simplex method, there is never a tie for the choice of the leaving variable, that is, at each iteration if x_k is the entering variable, there is exactly one basic variable x_r with $\bar{a}_{rk} > 0$ and $\bar{b}_r/\bar{a}_{rk} = \min(\bar{b}_i/\bar{a}_{ik} : \bar{a}_{ik} > 0)$.

(a) Can such a problem be degenerate? Explain.

(b) Can such a problem cycle? Explain.