C&O 350 Assignment 1, due Thursday, January 18, 1996

1. Later in the course, we will show that every l.p. problem has exactly one of the following properties: it has an optimal solution, it has no feasible solution, or it is unbounded. For each of the following problems, explain why it is not an l.p. problem, and show that it has none of the above properties:

   (a) minimize x_1 subject to $x_1 > 0$

   (b) minimize e^{-x_1} subject to $x_1 \geq 0$

2. (a) Convert to standard form:

$$\begin{aligned} \text{minimize} \quad -x_1 + x_2 & \\ \text{subject to} & \\ 2x_1 + x_2 &= 2 \\ x_1 &\geq 0 \end{aligned}$$

   (b) Convert to standard equality form:

$$\begin{aligned} \text{minimize} \quad -x_1 + x_2 & \\ \text{subject to} & \\ x_1 + x_2 &\geq 3 \\ 2x_1 + x_2 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$

3. Given (a_{ij}), $(i = 1, \ldots, m)$, $(j = 1, \ldots, n)$, and b_i, $(i = 1, \ldots, m)$, we want to find x_j, $(j = 1, \ldots, n)$, so that

$$\max_{i=1,\ldots,m} \left| \sum_{j=1}^{n} a_{ij} x_j - b_i \right|$$

is as small as possible. Why is this not (as it stands) a linear programming problem? Formulate an l.p. problem whose solution does solve the problem.

4. Where s is a parameter, consider the l.p. problem

$$\begin{aligned} \text{maximize } x_1 - 2x_2 & \\ \text{subject to} & \\ -x_1 + x_2 &\leq -2 \\ sx_1 - x_2 &\leq 0 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(a) For what values of s does the problem have a feasible solution? You may use geometry, but give an algebraic explanation too.

(b) For what values of s is the problem unbounded? You may use geometry, but try to give an algebraic explanation too.

5. Solve problem 2.1(a) of Chvátal by the simplex method.