C&O 350
Assignment 5

Due: Friday March 31 (§2, §3), Thursday March 30 (§1)

1. Consider the LP problem

$$(P)\ \max cx \text{ s.t. } Ax = b,\ x \geq 0,$$

where

$$A = \begin{bmatrix} 1 & 1 & \frac{1}{2} & 1 & 0 \\ 2 & 1 & 1 & 2 & -1 \\ 2 & 1 & 2 & 2 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 2 \\ 8 \end{bmatrix}, \quad c = \begin{bmatrix} 2 & 1 & 1 & -7 & 1 \end{bmatrix}.$$

Consider the basis $B = \{1, 3, 5\}$.

You are given the basic solution x^* corresponding to B, the solution y of $yA_B = c_B$, and the inverse of A_B:

$$A_B^{-1} = \begin{bmatrix} 3 & -\frac{1}{2} & -\frac{1}{2} \\ -4 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}, \quad y = \begin{bmatrix} 4 & -1 & 0 \end{bmatrix}, \quad x^* = \begin{bmatrix} 1 & 0 & 2 & 0 & 2 \end{bmatrix}.$$

You are also given that x^* is optimal for (P).

(a) Suppose that the value of c_2 in (P) changes from 1 to 4. How would you proceed to solve the new problem? Go as far as finding the entering and leaving variables on the first pivot.

(b) Suppose that the value of c_1 in (P) is changed from 2 to θ. For what values of θ is x^* optimal for the new LP?

(c) Suppose that a new variable x_6 is added to (P) with $c_6 = \theta$ and $A_6 = (-1, -4, 17)$. For what values of θ does B remain an optimal basis?

(d) Using the matrix description of dictionaries, write down the dictionary corresponding to the basis B.

(e) Suppose that the value of b_3 in (P) changes from 8 to 12. Solve the resulting LP.

2. A dairy wishes to produce milk containing exactly 4% butterfat, at least 5% protein, and at most 6% lactose. It can buy milk from three farms. Relevant information is tabulated below.

Farm	Fat(%)	Protein (%)	Lactose(%)	Cost/100lb
1	3	5	8	\$6.80
2	4	4	5	\$7.50
3	5	7	7	\$8.30

(a) Formulate an appropriate LP.

(b) Explain why there is no feasible solution composed only of milk from the three farms.

(c) Suppose that we can add water. What is the minimum percent of water that we can use to make an acceptable mix?

(d) Suppose that we can use all the water we wish. Formulate an appropriate LP.

(e) What is the minimal cost of 100 pounds of acceptable milk that can be mixed using the minimum possible amount of water?

3. Solve the following integer program using Gomory cuts.

$$\begin{array}{lrcrcrcl} \text{maximize} & 2x_1 & - & 4x_2 & + & x_3 & & \\ \text{subject to} & & & 2x_2 & + & 3x_3 & \le & 28, \\ & x_1 & - & x_2 & & & \le & 12, \\ & x_1 & , & x_2 & , & x_3 & \ge & 0, \text{ integers.} \end{array}$$

4. The following is a primal-feasible dictionary for a standard form linear program:

$$(D) \quad \begin{array}{rcrcll} x_r & = & \bar{b}_r & - & \sum_{s \notin B} \bar{a}_{rs} x_s & (r \in B) \\ z & = & \bar{d} & + & \sum_{s \notin B} \bar{c}_s x_s & \end{array}$$

For each of the following statements, say whether the statement is true or false, and justify your claim with a proof or a counterexample.

(a) If $\bar{c}_s \le 0$ for all $s \notin B$ then the basic feasible solution associated with (D) must be optimal for the primal problem.

(b) If $\bar{c}_s > 0$ for some $s \notin B$ then the basic feasible solution associated with (D) cannot be optimal for the primal problem.

(c) If $\bar{c}_s < 0$ for all $s \notin B$ then the primal linear program must have a unique optimal solution.

(d) If $\bar{c}_s \le 0$ for all $s \notin B$ and $\bar{c}_s = 0$ for some $s \notin B$ then the primal linear program must have more than one optimal solution.

(e) If (D) is nondegenerate and $\bar{c}_s \le 0$ for all $s \notin B$ then the dual problem must have a unique optimal solution.

(f) If (D) is nondegenerate and $\bar{c}_s > 0$ for some $s \notin B$ then the basic feasible solution associated with (D) cannot be optimal for the primal problem.