Class 1, C&O 466/666

1. Introduction (Chapter 1)
    1. Mathematical Formulation: example, level sets
    2. Definitions: local and global optima
    3. Definitions: convexity
2. Fundamentals of Unconstrained Optimization (Chapter 2)
    1. Example of nonlinear least squares data fitting
    2. Definitions: gradient, Hessian, Taylor's Theorem

Class 2

1. Fundamentals Unconstrained Opt. cont... (Chapter 2)
    1. Recognizing Solutions
        1. Definitions: gradient, Hessian, Taylor's Theorem, order notation (big and little O)
        2. directional derivative, curvature, linear model, direction of steepest descent
        3. first and second order necessary optimality conditions
        4. second order sufficient optimality conditions

Class 3, C&O 466/666

1. Overview of Algorithms
    1. Importance of convex functions and sets for global optima
    2. line search and trust region methods
        1. line search: steepest descent, Newton's method (scale invariance)

Class 4, C&O 466/666

1. Overview of Algorithms continued...
    1. quasi-Newton methods, trust region methods (comparisons)
    2. scaling
    3. rates of convergence

Class 5, C&O 466/666

1. Line Search Methods (Chapter 3), of type

$x_{k+1} = x_k + \alpha_k p_k$,

where p_k is the search direction and α_k is the step length

   1. Step Length
      1. Wolfe conditions (geometric interpretations)

         sufficient decrease - step is not too large curvature condition - step is not too small
      2. Lemma 3.1 (existence of step lengths)
   2. Convergence of Line Search Methods
      1. Theorem 3.2 (cos theta geometric interpretation)

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1. Line Search Methods continued ...(Chapter 3)
   1. Rate of Convergence
      1. Theorems 3.3,3.4 (linear convergence rate for steepest descent - dependence on condition number of Hessian)
      2. Theorem 3.5 (Dennis-More condition - quasi-Newton method Hessian approximate needs (asymptotic) convergence in search direction for superlinear convergence)
      3. Theorem 3.7 (with proof! - quadratic convergence of Newton's method near optimum.)

   Reference (for your interest only): CMP 1 789 601 (2001:03) 90C30 (65K05), Shi, Yixun(1-BLBS-CS), Globally convergent algorithms for unconstrained optimization. (English. English summary) Comput. Optim. Appl. 16 (2000), no. 3, 295--308

Class 7, C&O 466/666

1. Discussion of the derivative as a linear operator
2. Line Search Methods continued ...(Chapter 3)
   1. Theorem 3.7 (with proof! - quadratic convergence of Newton's method near optimum.)
3. Trust Region Methods (Chapter 4)
   1. basic outline,

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1. Trust Region Subproblem
   1. secular equation, optimality conditions, scaling, Cauchy point

Class 9, C&O 466/666

1. Conjugate Direction Methods
    1. Solving Ax=b for A pos. def. Expanding subspace theorem 5.2 with proof.

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1. Conjugate Gradient Methods
    1. Solving Ax=b for A pos. def. Practical form. Rates of convergence (clustering eigenvalues). Preconditioning.

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1. Conjugate Direction Methods cont...
    1. Gauss-Newton method for nonlinear least squares: properties of GN direction, Theorem 10.1, convergence rate

Class 12, C&O 466/666

1. Nonlinear Equations
    1. Newton's Method, derivation using linearization, Theorem 11.2 WITH PROOF, convergence rate, Inexact Newton method, merit functions
    2. Continuation (Homotopy) methods, (general outline only)

Class 13, C&O 466/666

1. Theory of Constrained Optimization, Chap. 12
    1. Definitions: constrained program, feasible set Omega, tangent cone of Omega at x, polar cone (dual cone).
    2. Geometric necessary conditions of optimality (extension of Fermat's theorem) for min f(x) x in Omega
    3. Geometric characterization of optimality for the convex case, min f(x) x in Omega , where f convex function and Omega convex set

Class 14, C&O 466/666

1. Theory of Constrained Optimization, Chap. 12, cont...
    1. Definitions: local vs global minimum, smoothing problems by adding constraints, active constraints,

Class 15, C&O 466/666

1.Theory of Constrained Optimization, Chap. 12, cont...
  (with basic convex analysis added)
    1.Basic Hyperplane Separation Theorem (with proof)
    2.Lemma: K is a closed convex cone if and only K=K++ (for derivation of Farkas' Lemma)
    3.Linearizing Cone, Cone of Gradients, Weakest Constraint Qualification, Karush-Kuhn-Tucker conditions

Class 16, C&O 466/666

1.Theory of Constrained Optimization, Chap. 12, cont...
  (with basic convex analysis/duality added)
    1.Basic separation theorem
    2.Lemma: K is a ccc iff K=K++ and proof using separation theorem
    3.derivation of Farkas' Lemma (using above lemma)

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1.Theory of Constrained Optimization, Chap. 12, cont...
  (with basic convex analysis/duality added)
    1.generalized Farkas' Lemma (with respect to cone ordering - with examples e.g. semidefinite programming)
    2.examples where Farkas' lemma fails

Class 18, C&O 466/666

1.Theory of Constrained Optimization, Chap. 12, cont...
  (with basic convex analysis/duality added)
    1.Lagrange Multiplier Theorem (with proof) of Generalized Convex Program
    2.dual of (generalized) convex programs (derivation using game theory)

Class 19, C&O 466/666

1.Theory of Constrained Optimization, Chap. 12, cont...
  (with basic convex analysis/duality added)
    1.Examples using KKT conditions
    2.Examples of duals of convex programs
    3.sensitivity interpretation of Lagrange multipliers for convex programs

Class 20, C&O 466/666

1. Optimality and Duality Continued...
    1. Examples of LP and convex QP duals in finite/infinite dimensions
       Comparison of examples in Hilbert and Banach spaces, i.e. Lagrange multiplier acts using an inner-product when the image of the constraint is an inner-product space; the action is still a bilinear form when the Lagrange multiplier is in the dual of a normed space. (The dual of C[0,1] is BV[0,1]. The optimal value in Example 3 is attained by a discontinuous function in (22), but the value can be approximated as closely as desired by a continuous function. This does not contradict the fact that C[0,1] is a Banach space.)

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1. Optimality and Duality Continued...
    1. Examples of duality gaps in convex programs
    2. First and Second order optimality conditions for general NLPs
2. Basics of Linear Programming

Class 22, C&O 466/666 - ('fixed point class on Mar 22')

1. Interior-Point Methods for Linear Programming
    1.
    2.
2.