UNIVERSITY OF WATERLOO
FINAL EXAMINATION
WINTER TERM 2003

Surname: ______________________________

First Name: ______________________________

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Course Number	C&O 367
Course Title	Nonlinear Programming
Instructor	Professor Henry Wolkowicz
Date of Exam	April 22, 2003
Time Period	9-12AM
Number of Exam Pages (including this cover sheet)	xx pages
Exam Type	Closed Book
Additional Materials Allowed	**no** calculators

Problem	Value	Mark Awarded
1	20	
2	20	
3	20	
4	15	
5	15	
6	10	
Total	100	

1

[20]

Consider the quadratic programming problem

$$\text{(QP)} \qquad \begin{array}{rrc} p^* := & \min & q(x) := \frac{1}{2}x^T Q x - c^T x \\ & \text{subject to} & Ax = b \end{array}$$

where A is $m \times n$, $Q = Q^T \succ 0$, and $Ax = b$ is consistent.

1. Prove that p^* is finite valued
2. Show that the optimum point exists and is unique.
3. Derive a dual quadratic (maximization) program to (QP) in m variables. State carefully what type of weak and strong duality results hold and why.

2

[20]

Consider the convex program

$$\text{(CP)} \qquad \begin{array}{rrc} p^* := & \min & f(x) := x_1^4 + x_2^4 \\ & \text{subject to} & g_1(x) = (x_1 - 1)^2 + (x_2 - 1)^2 - 2 \le 0 \\ & & g_2(x) = (x_1 + 1)^2 + (x_2 + 1)^2 - 2 \le 0 \end{array}$$

1. What is the feasible set F and the optimum point x^* of (CP)?
2. What is the tangent cone to F at x^*? What is the linearizing cone at x^*? (Identify the active set.) Verify whether or not these cones are equal.
3. Verify whether or not the Karush-Kuhn-Tucker (KKT) conditions hold at x^*. State why.

3

[20]

Let $f(x, y) = 2x^4 + y^2 - 4xy + 4y$ and let $(0\ 0)^t$ be the initial point.

1. Compute the first two iterates of the steepest descent method with exact line search for minimizing the function $f(x, y)$.
2. Compute the first two iterates of Newton's method for minimizing the function $f(x, y)$.
3. Compare the function values for the two algorithms at the iterates.

4

[15]

Let $f_i,\ i \in I$ be a collection of convex functions defined on a convex set Ω. Show that the function f defined by $f(x) = \sup_{i \in I} f_i(x)$ is convex on the region where it is finite. (You need to first show that the region where f is finite is a convex set.)

5

[15]

Suppose that C_1 and C_2 are convex sets in $\Re^n$ such that C_1 has interior points and C_2 does not contain any interior points of C_1. Prove that there is a hyperplane H in $\Re^n$ such that C_1 and C_2 lie in the opposite closed half-spaces determined by H, that is, there exist an $a \neq 0$ in $\Re^n$ and an $\alpha \in \Re$ such that

$$x \cdot a \leq \alpha \leq y \cdot a,$$

for all $x \in C_1$ and all $y \in C_2$. (You may use the basic hyperplane separation theorem and the hyperplane support theorem.)

6

[10]

Let A be an $m \times n$ matrix and let $b \in \Re^m$ be a fixed vector. Suppose the convex program

$$\begin{array}{rl} \min & \|x\|^2 \\ \text{subject to} & Ax \leq b \end{array}$$

is superconsistent and has solution x^*. Use the Karush-Kuhn-Tucker Theorem to show that there is a vector y in $\Re^m$ such that

$$x^* = A^T y \tag{1}$$

Do you need the superconsistent assumption to conclude (1)? Why?