C&O 367, Winter 2001
Assignment 2

Due on Tuesday, Feb. 4, (at start of class)
Instructor H. Wolkowicz

1. (10 marks) Let $f : \Re \to \Re$ be a convex function. Suppose that $a < b$.

   (a) Show that
   $$f(x) \leq \frac{b-x}{b-a} f(a) + \frac{x-a}{b-a} f(b), \quad \forall\ x \in [a,b].$$

   (b) Show that
   $$\frac{f(x)-f(a)}{x-a} \leq \frac{f(b)-f(a)}{b-a} \leq \frac{f(b)-f(x)}{b-x}, \quad \forall\ x \in [a,b].$$

   (c) Suppose that f is differentiable. Show that
   $$f'(a) \leq \frac{f(b)-f(a)}{b-a} \leq f'(b), \quad \forall\ x \in [a,b].$$

2. (10 marks) Suppose $f : \Re \to \Re$ is increasing and convex on its domain (a,b). Let g denote its inverse, i.e. the function with domain $[f(a), f(b)]$ and $g(f(x)) = x$ for all $a < x < b$. What can you say about convexity or concavity of g?

3. (10 marks) Suppose $f : \Re \to \Re$ is convex. Show that its *running average*, F, defined as
   $$F(x) = \frac{1}{x}\int_0^x f(t)dt, \quad x > 0,$$
   is convex. (You can assume that f is differentiable.)

4. (10 marks) Suppose $f : \Re \to \Re$ is convex and bounded above. Show that f is constant.

5. (15 marks) Suppose that

$$f(x) = b^t x + \frac{1}{2} x^t A x$$

where $b \in \Re^n$ and A is an $n \times n$ symmetric matrix. Show that $f(x)$ is bounded below on $\Re^n$ if and only if the minimum of f on $\Re^n$ is attained (i.e. there exists $\bar{x}$ such that $f(\bar{x}) = \min_{x \in \Re^n} f(x)$).