C&O 370 – DETERMINISTIC OR MODELS

Midterm Examination - Fall 1998

Friday, 6 November, 1998, 7.00 – 9.00pm **(2 hours)**

This examination consists of 4 questions and 9 pages.

ANSWER ALL QUESTIONS.

SHOW YOUR WORK.

Calculators permitted.

	Wolkowicz	TR 10:00-11:30

NAME: ____________________________

ID: ____________________________

SIGNATURE: ____________________________

For Marker Only	
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1 LINEAR PROGRAM MODEL, [25 marks]

Suppose you have just inherited \$6,000 and you want to invest it. Upon hearing this news, two different friends have offered you an opportunity to become a partner in two different entrepreneurial ventures, one planned by each friend. In both cases, this investment would involve expending some of your time next summer as well as putting up cash. Becoming a *full* partner in the first friend's venture would require an investment of \$5,000 and 400 hours, with an estimated profit to you of \$4,500. The corresponding figures for the second friend's venture are \$4,000 and 500 hours, with an estimated profit to you of \$4,500. However, both friends are flexible and would allow you to come in at any fraction of a full partnership you would like: your share of the profit would be proportional to this fraction.

Because you were looking for an interesting summer job anyway (maximum of 600 hours), you have decided to participate in one friend's or both friends' ventures in whichever combination would maximize your total estimated profit. You now need to solve the problem of finding the best combination.

1. Formulate a GAMS-like linear programming model for this problem.
2. Solve this model graphically. What is your total profit? What are the shadow prices?
3. What are the right-hand-side ranges?
4. Draw the graph of $z(b_1)$, i.e. graph the profit as the amount inherited changes.

1 continued...

Basic Variables	Current values	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_5	3	1	1	1	1	1	0	0
x_6	2	3	4	1	1	0	1	0
x_7	1	1	3	2	1	0	0	1
$(-z)$	0	-4	-6	-1	-3	0	0	0

Table 1: initial data tableau

Basic Variables	Current values	x_5	x_6	x_7
x_5	$\frac{12}{5}$	1	$-\frac{2}{5}$	$\frac{1}{5}$
x_1	$\frac{2}{5}$	0	$\frac{3}{5}$	$-\frac{4}{5}$
x_2	$\frac{1}{5}$	0	$-\frac{1}{5}$	$\frac{3}{5}$
$(-z)$	$\frac{14}{5}$	0	$\frac{6}{5}$	$\frac{2}{5}$

Table 2: current values

2 REVISED SIMPLEX METHOD, [25 marks]

The initial tableau, in canonical form, for a linear program to be minimized is given in Table 1.

A standard commercial linear-programming code employing the revised simplex method would have the following information at its disposal: (1) the initial data tableau in Table 1; (2) the current values of the basic variables and the row in which each is basic; and (3) the current coefficients of the initial unit columns. Items (2) and (3) are given in Table 2.

1. What is the current basic feasible solution?

2. We can define the *simplex multipliers* to be the shadow prices associated with the current basic solution even if the solution is not optimal. What are the values of the simplex multipliers associated with the current solution?

3. The current solution is optimal if the reduced costs of the nonbasic variables are nonnegative. Which variables are nonbasic? Determine the reduced cost of the nonbasic variables and show that the current solution is *not* optimal.

4. Suppose that variable x_4 should now be introduced into the basis. To determine the variable to drop from the basis, we use the minimum-ratio rule, which requires that we know not only the current right-hand-side values but also the coefficients of x_4 in the current tableau. These coefficients of x_4 need to be computed.

   In performing the simplex method, multiples of the initial tableau have been added to and subtracted from one another to produce the final tableau. the coefficients in the current tableau of the initial unit columns summarize these operations.

   (a) What multiple of rows 1,2, and 3, when added together, *must* produce the current row 1 (even though we do not know all the current coefficients in row 1)? The current row 2? The current row 3?

   (b) using the rational of 4a determine the coefficients of x_4 in the current tableau. Note that it is unnecessary to determine any of the other unknown columns.

   (c) How should the pivot operation be performed to update the tableau consisting of only x_5, x_6, and x_7? (Update the current values table).

2 continued...

Pattern \ Flight	1	2	3	4	5	6	Cost
1	-	x	-	x	-	-	1.40
2	x	-	-	-	-	x	0.96
3	-	x	-	x	x	-	1.52
4	-	x	-	-	x	x	1.60
5	x	-	x	-	-	x	1.32
6	-	-	x	-	x	-	1.12
7	-	-	-	x	-	x	0.84
8	x	-	x	x	-	-	1.54

Table 3: work patterns

3 [25 marks]

Air Anton is a small commuter airline running 6 flights per day from New York City to surrounding resort areas. Flight crews are all based in New York, working flights to various locations and then returning on the next flight home. Taking into account complex work rules and pay incentives, Air Anton schedulers have constructed the 8 possible work patterns detailed in table 3. Each row of the table marks the flights (with an x) that could be covered in a particular pattern and the daily cost per crew (in thousands of dollars). The company wants to choose a minimum total cost collection of work patterns that covers all flights exactly once.

Formulate this problem as a set partitioning ILP.

3 continued...

4 [25 marks]

A relief agency is urgently trying to get the maximum possible quantity of supplies from its base at Alto to the volcano-ravaged city of Epi. One available road goes via Billi. The agency estimates that the Alto-to-Billi part of that road can carry 500 tons per day, and the Billi-to-Epi segment, 320 tons. A second route goes via Chau and Domo, with the Alto-to-Chau part having capacity 650 tons, the Chau-to-Domo section, 470, and the Domo-to-Epi segment, 800. There is also a small mountain road with capacity 80 tons that connects Billi to Domo.

1. Formulate this problem as a maximum flow model by sketching the associated digraph. Indicate the source, the sink, and all capacities.
2. Solve your maximum flow problem by inspection. (Apply a Theorem to prove optimality, i.e. state the theorem you are using and how you used it.)
3. Show how to modify your digraph of part 1 to represent the model as a minimum cost network flow problem.

4 continued...