C&O 370 – DETERMINISTIC OR MODELS

Midterm Examination - Winter 1997

Friday, 28 February, 1997, 7.00 – 9.00pm **(2 hours)**

This examination consists of 4 questions and 9 pages.

ANSWER ALL QUESTIONS.

SHOW YOUR WORK.

Calculators permitted.

	Wolkowicz	TR 10:00-11:30

NAME: ______________________

ID: ______________________

SIGNATURE: ______________________

For Marker Only	
1	/
2	/
3	/
4	/
Total	/

Job Number	1	2	3	4	5
Brown	8	5	12	10	0
John	4	10	5	0	10
net income per unit	88	100	120	60	50

Table 1: Time, in hours, to complete a unit of job and the net income

1 LINEAR PROGRAM MODEL, [25 marks]

Drs Brown and John are highly qualified and experienced engineers. They set up a lucrative consultancy firm two years ago, each agreeing to make himself available 60 hours a week on consultancy duties. They could work on 5 different jobs, the details of which are in Table 1.

(For example, a unit of job 1 requires 8 hours of Brown's time, 4 hours of John's and will yield a net income of 88.)

1. Assuming that the number of units of each job that can be completed is limited only by the time available to the partners, formulate the problem of finding $x_i, i = 1, 2, 3, 4, 5$, that will maximize the total net income as an LP, where x_i is the number of units of job i to be performed. Show that $x^* = (5, 4, 0, 0, 0)$ is the optimum solution.

2. Each initially agreed to share the total income equally, but now Brown is claiming that he is entitled to more and is seeking to end the partnership unless he is given his due share. Suppose each agrees to accept his net contribution, in the LP sense, to the total net income,, how should it be divided? Show that under this scheme, John could still earn more by working jointly. Assume that John will be working on job 5 if the partnership ends.

3. There is another job, (job 6), that requires the service of another specialist Dr Ash. A unit of job 6 requires 7 hours each of Brown's and John's time and will yield a net income of 120, after paying off the consultancy fees to Ash. Determine whether total income can be increased by including job 6 in the optimal activities.

1 continued...

Basic Variables	Current values	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_5	3	1	1	1	1	1	0	0
x_6	2	3	4	1	1	0	1	0
x_7	1	1	3	2	1	0	0	1
$(-z)$	0	-4	-6	-1	-3	0	0	0

Table 2: initial data tableau

Basic Variables	Current values	x_5	x_6	x_7
x_5	$\frac{12}{5}$	1	$-\frac{2}{5}$	$\frac{1}{5}$
x_1	$\frac{2}{5}$	0	$\frac{3}{5}$	$-\frac{4}{5}$
x_2	$\frac{1}{5}$	0	$-\frac{1}{5}$	$\frac{3}{5}$
$(-z)$	$\frac{14}{5}$	0	$\frac{6}{5}$	$\frac{2}{5}$

Table 3: current values

2 REVISED SIMPLEX METHOD, [25 marks]

The initial tableau, in canonical form, for a linear program to be minimized is given in Table 2.

A standard commercial linear-programming code employing the revised simplex method would have the following information at its disposal: (1) the initial data tableau in Table 2; (2) the current values of the basic variables and the row in which each is basic; and (3) the current coefficients of the initial unit columns. Items (2) and (3) are given in Table 3.

1. What is the current basic feasible solution?

2. We can define the *simplex multipliers* to be the shadow prices associated with the current basic solution even if the solution is not optimal. What are the values of the simplex multipliers associated with the current solution?

3. The current solution is optimal if the reduced costs of the nonbasic variables are nonnegative. Which variables are nonbasic? Determine the reduced cost of the nonbasic variables and show that the current solution is *not optimal.*

4. *Suppose that variable x_4 should now be introduced into the basis. To determine the variable to drop from the basis, we use the minimum-ratio rule, which requires that we know not only the current right-hand-side values but also the coefficients of x_4 in the current tableau. These coefficients of x_4 need to be computed.*

   In performing the simplex method, multiples of the initial tableau have been added to and subtracted from one another to produce the final tableau. the coefficients in the current tableau of the initial unit columns summarize these operations.

   (a) *What multiple of rows 1,2, and 3, when added together,* must *produce the current row 1 (even though we do not know all the current coefficients in row 1)? The current row 2? The current row 3?*

   (b) *using the rational of 4a determine the coefficients of x_4 in the current tableau. Note that it is unnecessary to determine any of the other unknown columns.*

   (c) *How should the pivot operation be performed to update the tableau consisting of only $x_5, x_6,$ and x_7? (Update the current values table).*

2 continued...

3 [25 marks]

1. Consider the unconstrained minimization problem

$$\min_{w \in \Re^n} f(w)$$

   (a) State the first and second order necessary conditions for a local minimum.

   (b) State the first and second order sufficient conditions for a strict local minimum.

2. Consider the particular example

$$f(w) = f(x, y, z) = 2x^2 + xy + y^2 + yz + z^2 - 6x - 7y - 8z + 9.$$

   (a) Using the first order necessary conditions, find a minimum point of the function

   (b) Verify that the point is a relative minimum point by verifying that the second order sufficiency conditions hold.

   (c) Prove that the point is a global minimum point.

3 continued...

i \ j	1	2	3
0	8	18	31
1	-	10	21
2	-	-	12

Table 4: total net discounted cost

4 [25 marks]

At a small but growing airport, the local airline company is purchasing a new tractor for a tractor-trailer train to bring luggage to and from the airplanes. A new mechanized luggage system will be installed in 3 years so the tractor will not be needed after that. However, because it will receive heavy use, and maintenance costs are high, it may still be more economical to replace the tractor after 1 or 2 years. Table 4 gives the total net discounted cost associated with purchasing a tractor in year i and trading it in in year j (where year 0 is now).

The problem is to determine at what times the tractor should be replaced (if ever) to minimize the total costs for tractors.

1. Formulate this problem as a shortest path problem. (State the linear program and draw the network.)
2. Use Dijkstra's algorithm (discussed in class) to solve this problem. (Carefully show your steps.)
3. Write down the dual linear programming problem. Discuss an economic interpretation of the dual program.

4 continued...