C&O 370 – DETERMINISTIC OR MODELS

Midterm Examination - Fall 1997

Friday, 7 November, 1997, 7.00 – 9.00pm **(2 hours)**

This examination consists of 4 questions and 10 pages.

ANSWER ALL QUESTIONS.

SHOW YOUR WORK.

Calculators permitted.

01	☐	Wolkowicz	TR 10:00-11:30

NAME: ______________________

ID: ______________________

SIGNATURE: ______________________

For Marker Only	
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Basic Variables	Current Values	x_1	x_2	x_3	x_4	x_5
x_1	2	1		$\frac{1}{2}$	$-\frac{1}{2}$	
x_2	6		1	$\frac{1}{2}$	$\frac{1}{2}$	
x_5	4			$-\frac{1}{2}$	$\frac{1}{2}$	1
$(-z)$	-20			-2	-1	

Table 1: optimal tableau

1 SENSITIVITY ANALYSIS, [25 marks]

Consider the linear program

$$\begin{array}{lrcl}
\max & x_1 + 3x_2 & & \\
\text{subject to} & x_1 + x_2 & \leq & 8 \text{ (resource 1)} \\
& -x_1 + x_2 & \leq & 4 \text{ (resource 2)} \\
& x_1 & \leq & 6 \text{ (resource 3)} \\
& x_1, x_2 \geq 0 & &
\end{array}$$

1. Determine **graphically**:
   (a) the optimal solution;
   (b) the shadow prices on the three constraints;
   (c) the range on the objective coefficient of each variable, holding the coefficient of the other variable at its current value, for which the solution to part 1a remains optimal;
   (d) the range on the availability of each resource, holding the availability of the other resources at their current values, for which the shadow prices in part 1b remain optimal.

2. Answer the same questions using the the optimal tableau in Table 1 determined by the simplex method. You need only consider the ranges for the objective coefficients corresponding to the basic variable x_2 and the nonbasic variable x_3. You need only find the range for the availability of resource 1.

   Note: The matrix $B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ has inverse $B^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$

1 continued...

2 NETWORKS, [20 marks]

The general minimum-cost flow problem is given as follows:

$$\begin{array}{rl} \min & \sum_i \sum_j c_{ij} x_{ij} \\ \text{subject to} & \sum_j x_{ij} - \sum_k x_{kl} = b_i, \quad (i = 1, 2, \ldots n), \\ & \ell_{ij} \leq x_{ij} \leq u_{ij}. \end{array}$$

Assuming that the lower bounds on the variables are all finite, (this is $\ell_{ij} > -\infty$) show that any problem of this form can be converted to a transportation problem with lower bounds on the variables of zero; and with nonnegative, or infinite, upper bounds.

2 continued...

3 INTEGER PROGRAMMING [25 marks]

An airlines company is considering the purchase of new long-, medium-, and short-range jet passenger airplanes. The purchase price would be (in millions of dollars per airplane) 6.7, 5.0, 3.5, respectively. The Board of Directors has authorized a maximum commitment of \$150 million for these purchases. Regardless of which airplanes are purchased, air travel of all distances is expected to be sufficiently large enough so that these planes would be utilized at essentially maximum capacity. It is estimated that the net annual profit (after subtracting capital recovery costs) would be (in thousands of dollars per airplane) 420, 300, 230, respectively.

It is predicted that enough trained pilots will be available to the company to man 30 new airplanes. If only short-range planes were purchased, the maintenance facilities would be able to handle 40 new planes. However, each medium-range plane is equivalent to $1\frac{1}{3}$ short-range planes, and each long-range plane is equivalent to $1\frac{2}{3}$ short-range planes, in terms of their use of the maintenance facilities.

The information given above was obtained by a preliminary analysis of the problem. A more detailed analysis will be conducted subsequently. However, using the above data as a first approximation, management wisheds to know how many planes of each type should be purchased to maximize profit.

1. Formulate the integer linear programming model for this problem.
2. Formulate the problem with the additional constraint that at most two out of the three types of airplanes can be purchased.

3 continued...

	Grain	Floor space	Labour
Pigs	25 W or 20 C	25 sq.ft.	25 man hours
Chickens	10 W or 25 C	15 sq.ft.	40 man hours

Table 2: Requirements for animals, W = wheat, C = corn

4 LINEAR PROGRAM MODEL, [30 marks]

A farmer has 120 acres for growing wheat or corn. The yield for wheat is 55 bushels/acre/year, and that for corn is 95 bushels/acre/year. He can devote any fraction of the available 120 acres to wheat or corn. Cultivation needs 4 manhours of labour per acre per year, plus 0.15, 0.70 manhours per bushel of wheat, corn harvested, respectively. Unlimited quantities of wheat and corn can be sold to wholesalers at prices of \$1.75, \$0.95 per bushel, respectively. The farmer can also buy wheat and corn in the retail market at \$2.50 and \$1.50 per bushel, respectively,

The farmer also has a 10,000 sq.ft. facility for raising pigs and/or chickens. Pigs and chicken are sold at one year of age. A one year old pig sells for \$200; while one year old chicken (measured in units called a coop) sell for \$200 per coop. The requirements for raising (one unit of) pigs and chickens are given in Table 2.

The farmer already has 2000 manhours of labour devoted for raising pigs and/or chickens. He has another 2000 hours that he can devote to either raising animals or crops and he can obtain additional labour at \$4/hour; but for each hour of hired labour, he has to devote 0.1 hours of his own time for supervision.

1. Formulate an LP to determine the optimal crop/animal mix he should raise in order to maximize the net profit.

HINT: *You may want to consider several types of animals according to what type of grain (corn, wheat, bought, sold) they eat.*

4 continued...

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