C&O 370 – DETERMINISTIC OR MODELS

Final Examination - Fall 1997

DEC. 11, 2-5pm, PAC GYM AREA 6 **(3 hours)**

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This examination consists of 5 questions and 11 pages.

Each question is worth 20 points.

SHOW YOUR WORK.

Calculators permitted.

NAME: ______________________________

ID: ______________________________

SIGNATURE: ______________________________

	Regions		
# of Salespersons	A	B	C
1	35	21	28
2	48	42	41
3	70	56	63
4	89	70	75

Table 1: Estimated Increases in Sales

For Marker Only	
1	/
2	/
3	/
4	/
5	/
Total	/

1 [20 marks]

(Integer and Dynamic Programming) The sales manager for a publisher of college textbooks has six travelling salespeople to assign to three different regions of the country. She has decided that each region should be assigned at least one salesperson and that each individual salesperson should be restricted to one of the regions; but now she wants to determine how many salespeople should be assigned to the respective regions in order to maximize sales.

Table 1 gives the estimated increase in sales (in appropriate units) in each region if it were allocated various numbers of salespeople.

1. Model this problem as an Integer Programming Problem. (*GAMSlike*)

Assigned Station Located in Tract	Response Times Fire in Tract				
	1	2	3	4	5
1	5	12	30	20	15
2	20	4	15	10	25
3	15	20	6	15	12
4	25	15	25	4	10
5	10	25	15	12	5
Frequency of emergencies	2	1	3	1	3

Table 2: Average Response Time (in minutes)

2. Use dynamic programming to solve this problem. (Carefully define the recursion relationship.)
3. Show your work graphically, i.e. fill in a network to represent the problem.

1 continued...

2 [20 marks]

(Integer Programming) A new planned community is being developed, and one of the decisions to be made is where to locate the two fire stations that have been allocated to the community. For planning purposes, the community has been divided into five tracts, with no more than one fire station to be located in any given tract. Each station is to respond to *all* the fires that occur in the tract in which it is located as well as in the other tracts that are assigned to this station. Thus the decisions to be made consist of: (1) the tracts to receive a fire station; and (2) the assignment of each of the other tracts to one of the fire stations. The objective is: *to minimize the overall average of the response times to fires.*

Table 2 gives the average response time (in minutes) to a fire in each tract (the columns) if that tract is served by a station in a given tract (the rows). The bottom row gives the forecasted average number of fires that will occur in each of the tracts each day.

1. Formulate a complete integer programming model for this problem. Identify any constraints that correspond to mutually exclusive alternatives or contingent decisions.

2 continued...

3 [20 marks]

(Linear Programming) The Concrete Products Corporation has the capability of producing four types of concrete blocks. Each block must be subjected to four processes: batch mixing, mold vibrating, inspection, and yard drying. The plant manager desires to maximize profits during the next month. During the upcoming thirty days, he has 800 machine hours available on the batch mixer, 1000 hours on the mold vibrator, and 340 man-hours of inspection time. Yard-drying time is unconstrained. The production director has formulated his problem as a linear program with the following initial dictionary:

$$\begin{array}{rcrcrcrcrcr}
x_5 & = & 800 & - & x_1 & - & 2x_2 & - & 10x_3 & - & 16x_4 \\
x_6 & = & 1000 & - & 1.5x_1 & - & 2x_2 & - & 4x_3 & - & 5x_4 \\
x_7 & = & 340 & - & 0.5x_1 & - & 0.6x_2 & - & x_3 & - & 2x_4 \\
z & = & 0 & + & 8x_1 & + & 14x_2 & + & 30x_3 & + & 50x_4
\end{array}$$

where x_1, x_2, x_3, x_4 represent the number of pallets of the four types of blocks. The corresponding initial tableau is:

Basic variables	Current values	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_5	800	1	2	10	16	1		
x_6	1000	1.5	2	4	5		1	
x_7	340	0.5	0.6	1	2			1
$(-z)$	0	8	14	30	50			

After solving by the simplex method, the final tableau is

Basic variables	Current values	x_1	x_2	x_3	x_4	x_5	x_6	x_7
x_2	200		1	+11	19	1.5	-1	
x_1	400	1		-12	-22	-2	2	
x_7	20			0.4	1.6	0.1	-0.4	1
$(-z)$	6000			-28	-40	-5	-2	

a) By how much must the profit on a pallet of number 3 blocks be increased before it would be profitable to manufacture them?

b) What minimum profit on x_2 must be realized so that it remains in the production schedule?

c) If the 800 machine-hours capacity on the batch mixer is uncertain, for what range of machine hours will it remain feasible to produce blocks 1 and 2?

d) A competitor located next door has offered the manager additonal batch-mixing time at a rate of $4.00 per hour. Should he accept this offer?

e) The owner has approached the manager with a thought about producing a new type of concrete block that would require 4 hours of batch mixing, 4 hours of molding, and 1 hour of inspection per pallet. What should be the profit per pallet if block number 5 is to be included in the optimal schedule?

3 continued...

4 [20 marks]

(Network Programming) A catering company requires fresh linen napkins, which must be available at the start of each day. Normal laundry takes one full day at 1.5 cents per napkin; rapid laundry can be done overnight at 2.5 cents per napkin. The current napkin supply is 350 and there are 175 fresh napkins and 175 soiled napkins. It is known that the demand for the next three days is 300, 325, and 275 napkins respectively. It is against the health code to have dirty napkins linger overnight. All napkins after the third day can be sent to the normal laundry.

The caterer wants to decide how many napkins to send to each laundry service on each day so as to minimize cost but subject to meeting all his fresh napkin requirements and complying with the health code.

The decision variables for this problem are:

r_j = number of napkins sent to **rapid** laundry at the end of day $j,\ j = 0, 1, 2$.
n_j = number of napkins sent to **normal** laundry at the end of day $j,\ j = 0, 1, 2$.
u_j = number of fresh napkins (unused) at the end of day $j,\ j = 0, 1, 2$.

a) Formulate the problem as a linear program. You do **not** need to solve the LP.

b) Interpret the resulting model as a network-flow problem and draw the network.

c) Give an economic interpretation of the dual problem.

d) Suppose you could hold over the dirty napkins at no charge; how would your formulation of part b) change?

4 continued...

5 [20 marks]

(Integer Programming) Consider the pure integer linear programming problem

$$\text{Maximize} \quad z = 7x_1 + 3x_2$$

Subject to

$$\begin{aligned} 2x_1 + 5x_2 &\leq 30 \\ 8x_1 + 3x_2 &\leq 48 \\ x_1 \geq 0, x_2 &\geq 0 \quad \text{integral} \end{aligned}$$

a) Solve the relaxed LP problem to optimality. Show the optimal tableau. Note that both x_1 and x_2 are not integer.

b) Branch on variable x_1. Carry out the branching by adding constraints and applying the dual simplex method. Do not branch any further but outline the steps of a branch and bound strategy. In particular state the fathoming rules.

5 continued...