C&O 350 Assignment 5, due Tuesday, November 28, 1995

1. Prove that $Ax \leq b$ is unsolvable if and only if there exists a vector y such that $y \geq 0$, $yA = 0$, and $yb < 0$.

2. For the system $Ax = b,\ x \geq 0$, where $b = (3, 1, 2)^T$ and

$$A = \begin{pmatrix} 4 & 1 & 2 & 0 & -1 \\ 0 & 1 & 1 & -1 & 2 \\ 1 & 1 & 1 & -1 & 1 \end{pmatrix}$$

try to find a feasible solution by applying the revised simplex method using MATLAB to an auxiliary problem. Submit a computer printout of your work. If there is no feasible solution, find a vector y as in the Farkas Lemma.

3. Consider the linear programming problem (P) $\max cx$ subject to $Ax = b,\ x \geq 0$, where $b = (-2, 5, -1)^T$, $c = (2, -1, 0, -1, 1)$, and

$$A = \begin{pmatrix} -3 & 1 & 1 & 0 & 2 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & -2 & 0 & 0 & 1 \end{pmatrix}.$$

The feasible basis $B = (1, 2, 3)$ has corresponding basic feasible solution $x^* = (3, 2, 5, 0, 0)^T$.

(a) Find the solution y of $yA_B = c_B$, and use it to check that x^* is optimal.

(b) Suppose that c_5 in (P) is changed from 1 to a real number R. For what values of R is x^* still optimal?

(c) Suppose that R of part (b) is slightly too large to satisfy the conditions there. How would you solve the new problem? Go as far as finding the entering and leaving variables, and the new primal solution after one pivot.

(d) Suppose that c_2 in (P) is changed from -1 to a real number R. For what values of R is x^* still optimal?

(e) Suppose that a new variable x_6 with coefficients $a_{16} = 2$, $a_{26} = -3$, and $a_{36} = 4$ and $c_6 = R$ is added to the problem (P). For what values of R will x^* (with x_6^* defined to be zero) be optimal for the new problem?

(f) Suppose that the value of b_1 in (P) is changed from -2 to 3. Solve the resulting problem.

4. Consider the linear programming problem (Q) $\max cx$ subject to $Ax \leq b$, $x \geq 0$, where $b = (1,1,1)^T$, $c = (2,4,2,1)$, and

$$A = \begin{pmatrix} 9 & -1 & 2 & -3 \\ -14 & 3 & -3 & 5 \\ -6 & 1 & -1 & 2 \end{pmatrix}.$$

After adding slack variables and solving it is found that $B = (1,3,4)$ is an optimal basis, with corresponding optimal solution $x^* = (1,0,5,6,0,0,0)^T$. You are given that

$$A_B^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 4 & 3 & -1 \end{pmatrix}$$

(a) Give an optimal dictionary for (Q).

(b) Suppose that the value of b_3 is changed from 1 to a real number R. For what values of R does the optimal basis remain optimal?

(c) Suppose that R of part (b) is slightly too small to satisfy the conditions there. How should we proceed to solve the new problem? What do you conclude about the new problem?

(d) Suppose that a new constraint $x_1 + x_3 \leq 4$ is added to (Q). How should we proceed to solve the new problem? Go as far as identifying the entering and leaving variables on the first pivot.