C& O 350 **Assignment 4** **Due: November 14**

(1) Interpret the complementary slackness conditions in economic terms.

(2) Consider the LP in equality form:

$$\begin{array}{lrcl} \text{maximize} & \mathbf{cx} & & \\ \text{subject to} & \mathbf{Ax} & = & \mathbf{b} \\ & \mathbf{x} & \geq & \mathbf{0} \end{array}$$

where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 & 1 & 0 \\ 1 & 4 & 1 & 0 & 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \mathbf{c} = (2,1,1,1,1)$$

Using matrix formulas, compute the dictionary corresponding to the basis $B = (1,2)$.

(3) Consider the inequality LP:

$$\begin{array}{lrcccccccc} \max & z = & 6x_1 & + & 4x_2 & + & 2x_3 & & \\ \text{s.t.} & & 2x_1 & + & x_2 & + & x_3 & \leq & 4 \\ & & 4x_1 & + & 2x_2 & + & 2x_3 & \leq & 12 \\ & & 3x_1 & + & \frac{5}{2}x_2 & + & 2x_3 & \leq & 8 \\ & & x_1 & , & x_2 & , & x_3 & \geq & 0 \end{array}$$

The final dictionary obtained by applying the simplex method to solve this LP is presented below: (x_4, x_5, and x_6 represent the slack variables.)

$$\begin{array}{rcccccccc} x_1 & = & 1 & - & \frac{1}{4}x_3 & - & \frac{5}{4}x_4 & + & \frac{1}{2}x_6 \\ x_5 & = & 4 & & & + & 2x_4 & & \\ x_2 & = & 2 & - & \frac{1}{2}x_3 & + & \frac{3}{2}x_4 & - & x_6 \\ \hline z & = & 14 & - & \frac{3}{2}x_3 & - & \frac{3}{2}x_4 & - & x_6 \end{array}$$

For the basis $B = (1,5,2)$, what is the basis matrix A_B ? ¿From the above given dictionary, find

$$(i)\ \ \mathbf{A}_B^{-1},\ \ (ii)\ \ \mathbf{y} = \mathbf{c}_B\mathbf{A}_B^{-1},\ \ (iii)\ \ \overline{\mathbf{b}},\ \ \text{and}\ \ (iv)\ \ \overline{v}.$$

(4) Do problem 7.1 on page 116 of the textbook.

(5) Do problem 8.1 (b) on page 135 of the textbook. (In this problem, x_1, and x_2 are free variables. They can be eliminated as in the example given in the text on page 132.)

(6) Do problem 8.1 (e) on page 135 of the textbook. (Use matlab.)

(7) Consider the LP:

$$(P): \quad \max z = \mathbf{cx}, \quad \text{s.t.} \quad \mathbf{Ax} = \mathbf{b},$$

where no variable is required to be non-negative. Show that if (P) has an optimal solution, then, for all feasible solutions $\mathbf{x}$ of (P), $\mathbf{cx}$ is the same.

(HINT: If $\mathbf{x}_1$, and $\mathbf{x}_2$ are any two feasible solutions to (P), then, for any λ, $\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$ is also a feasible solution to (P), and if the value of z at $\mathbf{x}_1$ is v_1, and the value of z at $\mathbf{x}_2$ is v_2, then the value of z at $\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$ is $\lambda v_1 + (1-\lambda)v_2 = v_2 + \lambda(v_1 - v_2)$. If there are two different feasible solutions $\mathbf{x}_1$, and $\mathbf{x}_2$ at which the objective function values are different, show that (P) is unbounded.

Alternatively, note that the dual of (P) is (D) : $\min w = \mathbf{yb}$, s.t. $\mathbf{yA} = \mathbf{c}$. Show that if both the primal and dual are feasible, then, z has the same value at all primal feasible solutions.

Can you guess the condition on $\mathbf{c}$ under which the objective function value is the same for all feasible solutions?)