1 Phases I,II

Solve the following linear programs using the two phase approach.

1.

$$(LP)\quad \begin{array}{rrcrcll} \max & 8x_1 & + & 10x_2 & & & \\ \text{subject to} & x_1 & - & x_2 & = & 1 & \\ & x_1 & + & x_2 & \leq & 9 & \\ & x_1 & + & \frac{1}{2}x_2 & \geq & 4 & \\ & & & x_j & \geq & 0, & \forall j \end{array}$$

2.

$$(LP)\quad \begin{array}{rrcrcll} \max & 5x_1 & + & x_2 & & & \\ \text{subject to} & 2x_1 & + & x_2 & \geq & 5 & \\ & & & x_2 & \geq & 1 & \\ & 2x_1 & + & 3x_2 & \leq & 6 & \\ & & & x_j & \geq & 0, & \forall j \end{array}$$

2 BFS

Suppose that x^* is the only optimal solution of a linear programming problem in standard form. Prove that x^* is a basic solution.

3 Duality

Solve the linear program

$$(LP)\quad \begin{array}{rrcrcr} \min & 3y_1 & + & 6y_2 & & \\ \text{subject to} & 3y_1 & & & \geq & 3 \\ & -y_1 & + & y_2 & \geq & 1 \\ & 2y_1 & - & y_2 & \geq & -2 \\ & -2y_1 & + & y_2 & \geq & 1 \\ & y_1 & + & 2y_2 & \geq & -2 \\ & -y_1 & + & y_2 & \geq & 1 \\ & -3y_1 & + & 2y_2 & \geq & -3 \\ & y_1 & & & \geq & 0 \\ & & & y_2 & \geq & 0 \end{array}$$

(HINT: Do not be unduly upset by the number of constraints.)

4 Complementary Slackness

Using complementary slackness, check optimality of:

1.

$$x = \left(10\frac{1}{3}, 6\frac{1}{3}, 0\right)^t$$

2.

$$x = (7, 9, 0)^t$$

3.

$$x = (6, 7, 4)^t$$

for the following linear program.

$$(LP)\quad \begin{array}{rrcrcrcl} \max & 16x_1 & + & 21x_2 & + & 10x_3 & & \\ \text{subject to} & 3x_1 & + & 4x_2 & + & 2x_3 & \leq & 57 \\ & 2x_1 & + & x_2 & + & 2x_3 & \leq & 27 \\ & 4x_1 & + & 5x_2 & + & 4x_3 & \leq & 73 \\ & & & x_j & \geq & 0, & \forall j & \end{array}$$

5 Duality

Suppose that you have no knowledge of how to solve linear programs. But you do understand the duality theorem, as well as weak duality. Suppose further that you have access to a "black box" which, given a real $m \times n$ matrix A and m-vector b, solves the system of linear inqualities

$$(L) \quad Aw \leq b.$$

That is, it either gives an explicit vector w which solves (L), or says "(L) has no solution", if that is the case.
Explain how you would use this black box to solve the linear program

$$(LP)\quad \begin{array}{rrcrcl} \max & x_1 & + & x_2 & & \\ \text{subject to} & 2x_1 & - & x_2 & \leq & 3 \\ & 3x_1 & + & x_2 & \leq & 6 \\ & & & x_j & \geq & 0, \quad \forall j \end{array}$$

Specifically, determine the A and b which you would give the black box, and explain how to find a solution to (P) from a solution to (L). Generalize your result to any linear program.