(1) Formulate problem 1.6 in Chvátal as an LP.

(2) Picture the feasible region of the following LP:

$$\begin{array}{lrcrcrcr} \max & 3x_1 & + & 4x_2 & + & 6x_3 & & \\ \text{s.t.} & x_1 & + & 2x_2 & + & 4x_3 & = & 24 \\ & x_1 & , & x_2 & , & x_3 & \geq & 0 \end{array}$$

Guess the optimal solution without doing any elaborate calculations. (Hint: the optimal solution is at one of the 'corners' of the feasible region.)

(3) (a) Define precisely what it means to say that an LP is unbounded.
(b) Show that the following LP is unbounded as per your definition.

$$\begin{array}{lrcrcrcrcr} \max & & & & & 4x_3 & - & x_4 & & \\ \text{s.t.} & x_1 & & & - & x_3 & + & 9x_4 & = & 4 \\ & & + & x_2 & - & x_3 & + & 3x_4 & = & 3 \\ & x_1 & , & x_2 & , & x_3 & , & x_4 & \geq & 0 \end{array}$$

Hint: Note that x_1 and x_2 may be expressed in terms of x_3 and x_4. Find suitable feasible solutions by choosing appropriate values for x_3 and x_4, and computing the values of x_1 and x_2 based on them.

(4) Solve the LP in problem 2.1 (a) of Chvátal by the simplex method.

(5) Do problem 2 on page 12 of the handout.

(6) If $\mathbf{x}_1$ and $\mathbf{x}_2$ are two feasible solutions to an LP, and λ is any real number such that $0 \leq \lambda \leq 1$, then $\mathbf{x} = \lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2$ is also a feasible solution to the LP. (You may, if you like, assume that the given LP is an equality LP.)

(7)* For each of the following statements, either give a proof of the statement, or give a counter-example to show that the statement is false.

1. An LP is has no optimal solution, or has exactly one optimal solution, or has infinitely many different optimal solutions.
2. If the feasible region of an LP is unbounded, then the LP is unbounded.