1. (a) Value is 3:

Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. The result follows immediately upon substitution

$$\operatorname{Tr}(A) = a + d, \quad \operatorname{Tr}(A^2) = (a^2 + bc) + (bc + d^2), \quad \det(A) = ad - bc.$$

(b) Value is 2:

The matrix is triangular and so the determinant is the product of the diagonal, i.e. -60.

(c) Value is 2:

Expand the determinant along row 1 to get -1 times the determinant of the 3 by 3 matrix; the later can be found by expanding along row 1 again to get 36 as the final result.

(d) Value is 2:

The first two rows are linearly dependent; so the determinant has value equal to 0.

(e) Value is 3:

Elementary row operations (interchanging rows and keeping track of sign changes; adding a multiple of one row to another) yields determinant value equal to 24.

2. (a) Value is 2:

$$y = \frac{\det \begin{pmatrix} a & 1 \\ c & 0 \end{pmatrix}}{\det \begin{pmatrix} a & b \\ c & d \end{pmatrix}} = \frac{-c}{ad - bc}$$

-1 for wrong sign (in both parts a and b).

(b) Value is 3:

$$y = \frac{\det \begin{pmatrix} a & 1 & c \\ d & 0 & f \\ g & 0 & i \end{pmatrix}}{D} = \frac{fg - di}{D}$$

-1 for wrong sign (in both parts a and b).

(c) Value is 3:

There are many ways to solve this, e.g. set the determinant to 0 or use Gaussian elimination. Marks were given if clear details were given for the equivalence of linear dependence. Arithmetic errors were penalized but judgement was used on how clear the rest of the solution was, e.g. no marks were given for just stating the equivalence in terms of linear dependence and not getting started on the arithmetic.

Set

$$0 \;=\; \det\det \begin{pmatrix} 5 & -2 & 9 \\ -1 & 1 & 3 \\ 1 & -t & -2t \end{pmatrix}$$

5. value is 3:
   The eigenvalues are 1,1,4.

   value is 3:
   Eigenvectors can be found in the usual way, but may not be orthogonal

   value is 3:
   Orthogonal eigenvectors can be found in many ways, e.g.

$$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

   value is 3:
   The eigenvectors need to be normalized to find the orthogonal matrix for the diagonalization.

   Notes:
   7/12 if eigenvectors were not normalized and not orthogonal
   8/12 if eigenvectors were normalized but not orthogonal
   10/12 if eigenvectors were orthogonal but not normalized
   -1 if the diagonal matrix was missing or incorrect.