$x_1 >= 0.55(x_1 + x_2)$

i.e. first mortgages >= 0.55(total mortgage lending) and also

$x_1 >= 0.25(x_1 + x_2 + x_3 + x_4)$

i.e. first mortgages >= 0.25(total loans)

(c) policy condition 2

$x_2 <= 0.25(x_1 + x_2 + x_3 + x_4)$

(d) policy condition 3 - we know that the total annual interest is $0.14x_1 + 0.20x_2 + 0.20x_3 + 0.10x_4$ on total loans of $(x_1 + x_2 + x_3 + x_4)$. Hence the constraint relating to policy condition (3) is

$0.14x_1 + 0.20x_2 + 0.20x_3 + 0.10x_4 <= 0.15(x_1 + x_2 + x_3 + x_4)$

Note: whilst many of the constraints given above could be simplified by collecting together terms this is not strictly necessary until we come to solve the problem numerically and does tend to obscure the meaning of the constraints.

Objective

To maximise interest income (which is given above) i.e.

maximise $0.14x_1 + 0.20x_2 + 0.20x_3 + 0.10x_4$

In case you are interested the optimal solution to this LP (solved using the package as dealt with later) is x_1= 208.33, x_2=41.67 and x_3=x_4=0. Note here this this optimal solution is not unique - other variable values, e.g. x_1= 62.50, x_2=0, x_3=100 and x_4=87.50 also satisfy all the constraints and have exactly the same (maximum) solution value of 37.5

Blending problem

Consider the example of a manufacturer of animal feed who is producing feed mix for dairy cattle. In our simple example the feed mix contains two active ingredients and a filler to provide bulk. One kg of feed mix must contain a minimum quantity of each of four nutrients as below:

Nutrient	A	B	C	D
gram	90	50	20	2

The ingredients have the following nutrient values and cost

	A	B	C	D	Cost/kg
Ingredient 1 (gram/kg)	100	80	40	10	40
Ingredient 2 (gram/kg)	200	150	20	-	60

What should be the amounts of active ingredients and filler in one kg of feed mix?

Blending problem solution

Variables

In order to solve this problem it is best to think in terms of one kilogram of feed mix. That kilogram is made up of three parts - ingredient 1, ingredient 2 and filler so let:

x_1 = amount (kg) of ingredient 1 in one kg of feed mix
x_2 = amount (kg) of ingredient 2 in one kg of feed mix
x_3 = amount (kg) of filler in one kg of feed mix
where $x_1 >= 0$, $x_2 >= 0$ and $x_3 >= 0$.

Essentially these variables (x_1, x_2 and x_3) can be thought of as the recipe telling us how to make up one kilogram of feed mix.

Constraints

- nutrient constraints

$100x_1 + 200x_2 >= 90$ (nutrient A)
$80x_1 + 150x_2 >= 50$ (nutrient B)
$40x_1 + 20x_2 >= 20$ (nutrient C)
$10x_1 >= 2$ (nutrient D)

Note the use of an inequality rather than an equality in these constraints, following the rule we put forward in the Two Mines example, where we assume that the nutrient levels we want are lower limits on the amount of nutrient in one kg of feed mix.

- balancing constraint (an *implicit* constraint due to the definition of the variables)

$x_1 + x_2 + x_3 = 1$

Objective

Presumably to minimise cost, i.e.

minimise $40x_1 + 60x_2$

which gives us our complete LP model for the blending problem.

In case you are interested the optimal solution to this LP (solved using the package as dealt with later) is x_1= 0.3667, x_2=0.2667 and x_3=0.3667 to four decimal places.

Obvious extensions/uses for this LP model include:

- increasing the number of nutrients considered
- increasing the number of possible ingredients considered - more ingredients can never increase the overall cost (other things being unchanged), and may lead to a decrease in overall cost
- placing both upper and lower limits on nutrients
- dealing with cost changes
- dealing with supply difficulties
- filler cost

Blending problems of this type were, in fact, some of the earliest applications of LP (for human nutrition during rationing) and are still widely used in the production of animal feedstuffs.
