C&O 370 Deterministic OR Models – W11

Assignment 2 Due Monday Feb. 7, 2011

Assignments are due at the start (9:30-ϵAM) of class on the due date, or under the door MC6065 by midnight-ϵ before the due date.

Contents

1 LP Model - and Solve using AMPL —— 15 Marks

50 students have been admitted to University of Waterloo. (They are numbered $S_1, \ldots, S_{50}$.) Each one of the 50 students is required to take courses; the number and type of program are restricted by their program of study, A to D. The number of students admitted to each program is given in Table 1. The number of courses that each program requires their students

program of study	A	B	C	D
Number of Students	17	9	12	12

Table 1:

to take is given in Table 2. The available spots for taking courses in each program is given in

program of study	A	B	C	D
Number of courses required	5	3	2	4

Table 2:

Table 3. A student of one program can only take courses in certain programs, as indicated

program of study	A	B	C	D
Spots in Courses Available	81	39	28	59

Table 3:

in Table 4. The cost of a course in a program depends on which program the student is in. The (units of) costs for a student from one program taking a course in a specific program are given in Table 5.

1. Formulate the problem of finding a choice of students to courses in programs while ignoring the cost.

2. Formulate the problem of finding a choice of students to courses in programs so as to minimize the total cost.

3. Solve with AMPL. (Write a general model, i.e. use sets and parameters. You can use *subsets* as well. For an example see page 10 in www.unc.edu/~pataki/hhh/ampl/amplhandout.pdf.)

4. Suppose that the students enter the preferences for their courses. For example, a student in program B enters the preferences: B,B,D; while a student in program A enters the preferences: A,D,D,C,C. Formulate a model that takes the student preferences into account. (Can you include the total costs as well?)

Student Program	A	B	C	D
Acceptable Course Program Types	A,C	B,C	C	A,B,C,D

Table 4:

student \ course	A	B	C	D
A	2	0	5	0
B	0	3	7	0
C	0	0	1	0
D	3	5	2	0

Table 5:

2 Elementary Sensitivity Analysis Review —— 10 Marks

Consider the primal linear program (LP)

$$(P) \qquad \begin{array}{rrl} \max & 3x_1 + x_2 & \\ \text{s.t.} & x_1 + x_2 & \leq 4 \\ & 2x_1 + x_2 & \leq 6 \\ & x_1 + 2x_2 & \leq 6 \\ & x_1 + x_2 & \geq 1 \\ & x_1, x_2 & \geq 0 \end{array}$$

1. (Duality and Game Theory) Formulate the primal LP (P) as the optimal, worst case, scenario for player X, i.e. form a payoff function as done in class and consider the corresponding max min problem. Then derive the dual using the min max strategy for the player Y. Show how *weak duality* follows from this derivation. Show that complementary slackness is equivalent to strong duality.

2. Using the matrix approach to sensitivity analysis, calculate the range on perturbations of the objective coefficients (individually), so that the optimal basis remains optimal.

3. Let z^* be the optimal value as a function of the perturbations of the objective coefficients. Give a formula for z^* for perturbations that leave the basis optimal.

4. Sketch the region of simultaneous perturbations that leave the optimal basis as optimal.

5. Calculate the dual solution and directly check that it is feasible for the dual linear program.

3 Blending Problem with Sensitivity Analysis —— 10 Marks

Shellco Inc produces three grades of gasoline (Premium, Midgrade, and Economy) from 3 types of crude oil (A,B, and C).

The minimum octane level for the grades of gasoline are:
93 for Premium, 89 for Midgrade, and 86 for Economy.
The maximum sulfur levels for the grades of gasoline are:
2.25% for Premium, 2.40% for Midgrade, 2.45% for Economy.
The revenue for the grades of gasoline are:
$25 for Premium, $30 for Midgrade, $30 for Economy.

The octane levels of the crude oil are:
97 for crude A, 89 for crude B, 82 for crude C.
The sulphur levels of the crude oil are: 2.5% for crude A, 1.5% for crude B, .75% for crude C.
The costs per barrel for the crude oil are:
$10 for crude A, $15 for crude B, $18 for crude C.
Crude can be purchased up to the following amounts:
5000 barrels of crude A, 5000 barrels of crude B, 7000 barrels of crude C.

It is desired to meet the following expected demand for the 3 gasolines: 2000 barrels of Premium, 4000 barrels of Midgrade, 6000 barrels of Economy.

1. Formulate Shellco's problem of maximizing the net profit as a linear program. Solve the LP with AMPL. (Model the problem as a general problem, i.e. use sets and parameters.)

2. Use AMPL to calculate the *gradient*[1] of the optimal objective value (with respect to perturbations in the supplies and demands.[2] Note that there is no AMPL command that will just give you the gradient - but you can use AMPL to solve several programs with perturbed right-hand-sides to gather enough information to get your answer. Where possible, interpret the entries in the gradient in economic terms.

[1] See Appendix A in the class notes for a Multivariate Calculus Review.

[2] The gradient is the vector of derivatives of the optimal objective value with respect to each supply and demand constant.

4 Ranges —— 15 Marks

A chemical company produces two products. Product1, known as *StripEasy*, is a paint and finish remover for tough refinishing jobs (and is quite thick). Product 2, sold under the brand name *RenewIt* is less viscous and intended for easier refinishing (or to be used after one application of StripEasy). Every can of StripEasy returns a profit of \$0.50, and the returns for RenewIt is \$0.30 per can.

Because these products can only be shipped out at the end of each week, the total amount produced must fit within the company warehouse, which has a capacity of $400,000$ cubic feet. Each container (for either product) consumes 2 cubic feet of warehouse space.

Both products are produced through the same system of pipes, vessels, and processors. Production rates are 2000 and 3000 cans per hour for StripEasy and RenewIt, respectively, with a total of 120 hours per week available.

Marketing has determined that under the present market state and advertising level, the maximum weekly amounts that can be sold are $250,000$ cans of StripEasy and $300,000$ cans of RenewIt. Finally, because of a previous contract, the company must furnish at least $60,000$ cans per week of RenewIt to a particular customer.

The problem is to maximize weekly profit subject to given constraints.

1. Formulate this problem as a linear program. Then, model and solve it using AMPL.

2. Write down the optimal tableau, the corresponding optimal basis matrix A_B, its inverse A_B^{-1}, and an optimal dual solution.

3. Is the primal optimal solution unique? Is the dual optimal solution unique? (and why?)

4. The simplex tableau contains information for the organization on how to grow, increase profits, etc... You can use AMPL to help answer the following questions.

   (a) What is a reasonable price to pay for addition warehouse space?

   (b) What is a reasonable penalty to pay for not completely satisying the contract for $60,000$ cans of RenewIt?

   (c) find the ranges for the right-hand sides of the constraints.

   (d) find the ranges for the coefficients in the objective function.