C&O 370 Deterministic OR Models – Winter 2011

Assignment 5 Due date: Friday Apr. 1, 2011

Assignment 5 is due before the start (9:30-ϵAM) of the class.
Write your name and ID# clearly, and underline your last name.
Important: This assignment is not *meant* to be a joke.

Contents

1 Dynamic Programming —— 10 Marks **2**

2 Gomory Cuts —— 9 Marks **2**

3 Network Flows —— 10 Marks **2**

4 Branch and Bound —— 5 Marks **3**

5 Bonus: Dynamic Programming —— 10 Marks **3**

6 Suggested Problems - Do NOT Hand in —— **4**

1 Dynamic Programming —— 10 Marks

Consider the investment problem with 6 different investments. Only complete numbers of guaranteed investments, GICs, are possible. There are 55[1] units available to invest ($2200 per unit). At least two GICs of each type must be chosen. The following Table 1 provides the data.

Investment type	Cost (units/certificate)	r=1	2	3	4	5
1	4	2	3	8	16	23
2	3	1	2	4	7	12
3	3	4	8	15	24	30
4	4	4	9	23	36	42
5	5	5	11	28	39	45
6	7	6	12	33	46	49

Table 1: Return when buying r GICs

1. Use dynamic programming to choose the number of GICs to maximize return.
2. Model and solve the problem using AMPL.

2 Gomory Cuts —— 9 Marks

Pages 114-115, Exercise 6.11.

3 Network Flows —— 10 Marks

Consider the minimum-cost flow problem in Figure 1. (This problem has five nodes with a supply of 20 at node 1 and a demand of 5 and 15 at nodes 4 and 5, respectively. The arc capacity and arc costs are given in the text boxes adjacent to the arcs. A value of ∞ indicates that there is no arc capacity.)

1. Model and solve using Dijkstra's dynamic programming model (e.g. the one presented in class).
2. Model and solve using AMPL.

[1]Note: this changed from 45 to 65 units, which resulted in an infeasible problem.

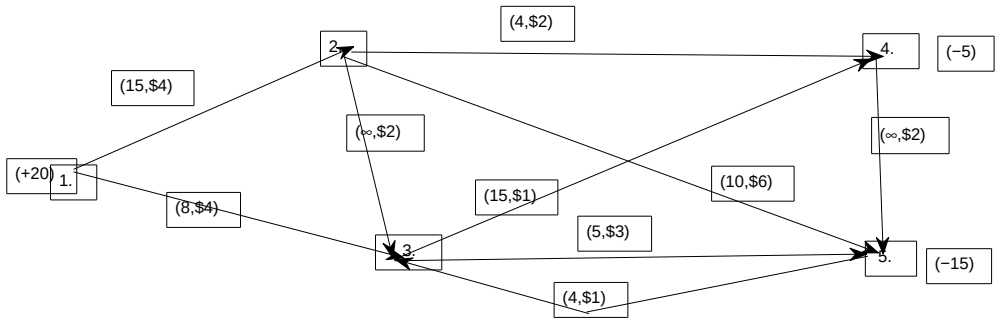

Figure 1: Minimum-cost flow problem

4 Branch and Bound —— 5 Marks

On Page 113, Exercise 6.8: Are the optimal values $z = 90$ in L_1, $z = 80$ in L_5, $z = 70$ in L_6, correct or wrong. What can you say about the optimal value in the general case? (Provide a proof.)

5 Bonus: Dynamic Programming —— 10 Marks

1. An army Colonel wants to send a tank across a desert that is 1,027 KM wide. The tank is able to travel 5 KM/gallon. However, once the tank enters the desert, there is no additional gasoline available. The tank has a carrying capacity of 100 gallons.

   The Colonel plans to fill up the tank at the depot at the beginning of the desert, and then drive into the desert to a *first temporary gas dump*, where some gasoline is unloaded and stored; and then the Colonel plans on returning to the depot to fill up on gasoline again. This process is continued until enough gas is stored at the dump so that it is the new gasoline *depot* so that the tank can continue past it into the desert.

   Formulate the problem of crossing the desert using the minimum quantity of gasoline as a *dynamic programming problem*. Solve the problem and provide the optimal strategy.

2. What is the best solution if the Colonel decides to send a tank across the desert every day for a month?

3. What is the best solution if the Colonel plans to send 10 tanks across the desert?

4. Does/did the Colonel succeed?

6 Suggested Problems - Do NOT Hand in

Pages 109-112, Exercises 6.1 and 6.6.