

C&O370/CM443 Deterministic OR Models – Winter 2011

Assignment 4

Due date: Wed. Mar. 16, 2011

Assignments are due at the start of class on the due date.
Write your name and ID# clearly, and underline your last name.

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1 Scheduling

– 10 Marks

There are eight group members who are working to complete a project. They have different qualifications and so are assigned different tasks. During the month, there are seven meetings planned; each meeting requires the attendance of some of the group members as follows in Table 1.

Meeting number	group members required to attend
meeting 1	1, 4, 6
meeting 2	2, 5, 4, 7
meeting 3	3, 8, 1, 2
meeting 4	2, 3, 5
meeting 5	7, 8
meeting 6	3, 4, 5
meeting 7	1, 6, 8

Table 1: Attendance needs

There are many suitable rooms to hold meetings. Each meeting takes up one half of a day. Meetings can be held concurrently if they do not require a common group member.

1. Formulate a mathematical model and an AMPL model for finding the maximum number of these meetings that can be scheduled in one day. Find the optimal solution using AMPL
2. Formulate a mathematical model and an AMPL model for finding the minimum total time period needed to hold all the meetings. Solve using AMPL.
3. Suppose that the meetings had the following rules/priorities:

meeting 3 can only proceed after meeting 2 is finished

meeting 5 can only proceed after meeting 2 is finished

meeting 6 can only proceed after meeting 3 is finished

meeting 7 can only proceed after meeting 5 is finished

Resolve Item 2 while maintaining these rules.

4. Generate your own table of data for 50 meetings, 20 group members, and 2 – 6 group members attending each meeting.¹ Repeat Items 1, 2 with this new data. (Keep track of computer solution times.)

¹That is, generate the vector of integers $numbers \in \mathbb{R}^{50}$ with elements in $\{2, 3, 4, 5, 6\}$ for the number of attendees at each meeting. Then for each meeting generate $numbers_i$ random integers that indicate the group members required to attend that meeting.

2 Merchant of Venice and $\{0, 1\}$ Programming - 10 Marks

From Shakespeares play *The Merchant of Venice*, 1597: Portia is a law graduate with an obsession for highly intelligent boys. Her sole ambition is to marry an extremely intelligent boy. She devises a clever scheme to choose her mate. She has three caskets: of gold, silver, lead. And, she hides a beautiful portrait of herself in one of them. The suitors are asked to figure out which casket contains the portrait. If his choice is correct, he gets to marry Portia. He can only do this test once. Portia puts inscriptions on the three caskets:

gold The portrait is in this casket.

silver The portrait is not in this casket.

lead The portrait is not in the gold casket.

She states that at most one of the three inscriptions is true.

1. Model the problem of identifying the casket containing the portrait as a 0-1 integer feasibility problem.
2. Attempt to solve the model in Item 1 using an LP relaxation. Is the solution from the LP integer valued? If not, is the closest integer feasible solution the optimal solution?
3. Solve the model in Item 1 using AMPL.

3 Facility Location - 10 Marks (+10BONUS)

A city is divided into 8 zones. The best location for a bank for each zone has been determined. Table 3 gives estimates for the average number of minutes it takes for a person from zone j

	Average driving time							
between i/and j	1	2	3	4	5	6	7	8
1	10		25		40			30
2		8	60	35		60	20	
3	30		5	15	30	60	20	
4	25		30	15	30	60	25	
5	40		60	35	10		32	23
6		50	40	70		20		25
7	60	20		20	35		14	24
8	30		25		25	30	25	9

Table 2: Average driving time for person in zone j to use bank in zone i .

to use a bank located in zone i . Due to traffic, the matrix is not symmetric. Blank spaces indicate that average times are greater than 100.

1. Determine a set covering model to find the zones for the minimum number of banks under the restriction that banks should be accessible within an average of 25 minutes driving time. Solve the LP relaxation and then solve using AMPL. Compare the solutions. (Repeat with accessible driving time of 35 minutes.)
2. Now suppose that there are 8 different banks, labelled 1, 2, 3, 4, 5, 6, 7, 8, that want to locate at the locations in these * zones. The banks interact with each other and would like to minimize the total sum of all miles that are driven by people interacting between all the banks. Table 3 contains the number of people that are expected to interact between the banks. Model the problem of deciding which banks go into which zone so as to minimize the total driving time. Try to solve a relaxation of this problem? Try to solve the problem using AMPL.

	Average number of people							
between i/and j	1	2	3	4	5	6	7	8
1	0	26	13	28	15	13	13	13
2	26	0	10	17	19	11	17	11
3	13	10	0	10	7	8	17	7
4	28	17	10	0	12	20	13	12
5	15	19	7	12	0	15	23	27
6	13	11	8	20	15	0	6	5
7	13	17	17	13	23	6	0	17
8	13	11	7	12	27	5	17	0

Table 3: Average number of people driving between bank i and bank j .