C&O370/CM443 Deterministic OR Models – Winter 2011

Assignment 1 Due date: Friday Jan. 21, 2011

Assignments are due at the start of class on the due date.
Write your name and ID# clearly, and underline your last name.

Contents

1 Problem 1: LP Review/Duality/AMPL – 10 Marks

Consider the *primal* linear programming problem, (P), given in AMPL form. (The file with the AMPL problem is also available here: examp1.mod.)

```
var x1;
var x2;
var x3;
var x4;
var x5;
var x6;
var x7;
var x8;
minimize Expense:  +(-6)*x1+(-9)*x2+(12)*x3+(-14)*x4+(-23)*x5+(5)*x6+(-18)*x7 +(-2)*x8;
subject to T1:   +(-3)*x1+(5)*x2+(3)*x3+(4)*x4+(-2)*x5+(2)*x6+(9)*x7 <= -6;
subject to T2:   +(5)*x1+(2)*x2+(15)*x3+(-5)*x4+(6)*x5+(3)*x6+(6)*x7 -x8= 2;
subject to T3:   +(3)*x1+(0)*x2+(-2)*x3+(3)*x4+(10)*x5+(0)*x6+(4)*x7 <= -44;
subject to T4:   +(-3)*x1+(6)*x2+(-1)*x3+(5)*x4+(-7)*x5+(-5)*x6+(6)*x7 +(3)*x8 <= -9;
subject to xlimit1:  x1 >=0;
subject to xlimit2:  x2 >=0;
subject to xlimit3:  x3 >=0;
subject to xlimit4:  x4 <=0;
subject to xlimit5:  x5 <=0;
subject to xlimit6:  x6 >=0;
subject to xlimit7:  x7 <=0;
```

1. Write down the dual problem (D) of (P). (In the dual problem, the constraints should be a mixture of $\leq, \geq, =$ constraints. The variables should be a mixture of $\leq, \geq$, and free.)

2. Write down all the complementary slackness conditions for (P),(D).

3. Consider the following (approximate) possible vector of solutions for (P).

$$x = \begin{pmatrix} 56.182583862749446 \\ 49.841291931374272 \\ 0.000000000000052 \\ -0.000000000000128 \\ -0.000000000000128 \\ 0.000000000000344 \\ -53.136937897062069 \\ ? \end{pmatrix}^T,$$

where the last component $x8$ is not given. First, use the given data and find an appropriate $x8$ so that the resulting vector provides a (approximate) feasible solution to (P). (Show your work carefully.) Then, use part 2 to find a (approximate) solution to (D) and also, to show (provide the details) whether or not the vector x above is optimal for (P).

4. Is the dual optimal solution unique? Is the primal optimal solution unique? (Explain carefully why or why not. If the solution (primal or dual) is not unique, then provide an alternate (primal or dual, resp.) solution.)

2 LP Formulation and AMPL; a Transportation Problem - 10 Marks

Suppose that there are two canning plants (at Halifax, Winnipeg) and three markets (at Montreal, Toronto, Vancouver). Table 1 provides the data; shipping distances are in thousands of KM, shipping costs are assumed to be $90.00 per case per thousand KM, and supplies (and demands) are in numbers of cases.

	Markets			
	Montreal	Toronto	Vancouver	
Plants	Shipping Distances			Supplies
Halifax	1.244	1.790	6.145	350
Winnipeg	2.754	2.224	2.293	600
Demands	325	300	275	

Table 1: shipping data for Problem 2

1. Formulate an LP problem for minimizing the transportation cost while meeting customer demand and satisfying the supply constraints. (Your solution should include a description of the sets, the main decision variables, and the constraints.)

2. Solve the LP using the AMPL software. Submit a printed version of your <u>LP model</u> (including any data files), and a log of your session on AMPL that shows (i) the optimal value, and (ii) an optimal solution.

3. Repeat part 2 but replace the first number of supplies (350) and the first number of demands (325) using the two three digit numbers formed from: the first 3 digits of your student ID, and the second 3 digits of your student ID. The larger of these two numbers replaces the supply and the smaller number replaces the demand.

3 LP Formulation – Manufacturing —— 10 Marks

A liquor company produces and sells two kinds of liquor: blended whiskey and bourbon. The company purchases intermediate products in bulk, purifies them by repeated distillation, mixes them, and bottles the final product under their own brand names. In the past, the firm has always been able to sell all that it produced. The firm has been limited by its machine capacity and available cash. The bourbon requires 3 machine hours per bottle while, due to additional blending requirements, the blended whiskey requires 4 hours of machine time per bottle. There are 20,000 machine hours available in the current production period. The direct operating costs, which are principally for labor and materials, are $3.00 per bottle of bourbon and $2.00 per bottle of blended whiskey.

The working capital available to finance labor and material is $4000; however, 45% of the bourbon sales revenues and 30% of the blended-whiskey sales revenues from production in the current period will be collected during the current period and be available to finance operations. The selling price to the distributor is $6 per bottle of bourbon and $5.40 per bottle of blended whiskey.

1. Formulate a linear program that maximizes contribution subject to limitations on machine capacity and working capital. (Your solution should include a description of the sets, the main decision variables, and the constraints.)

2. What is the optimal production mix to schedule?

3. Can the selling prices change without changing the optimal production mix?

4. Suppose that the company could spend $400 to repair some machinery and increase its available machine hours by 2000 hours. Should the investment be made? What interest rate could the company afford to pay to borrow funds to finance its operations during the current period?