C&O 367/CM 442 Nonlinear Optimization – Winter 2009

Assignment 1 Due date: Wednesday Jan. 14, 2009

Assignments are due before the start of class on the due date.
Write your name and ID# clearly, and underline your last name.

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C&O 367
Assignment 1

Due on Thursday, Jan. 14 (before start of class)
Instructor H. Wolkowicz

1 Introduction to MATLAB

MATLAB; see saw.uwaterloo.ca/matlab/ For the MATLAB output, you can use the **diary** command, i.e. enter the command: **diary filename.txt** before the MATLAB session; and enter the command: **diary off** just before the end of the session. You can then edit and submit the file filename.txt.

1. Enter the command matlab to start up the MATLAB session. Then enter

   help optim

   You will see a list of the optimization functions that are available with the optimization toolkit.

2. Now try the command

   demo toolbox optim

   This will bring up a menu. You can try several choices. One choice is a **Tutorial for the Optimization Toolbox**. This demonstrates several of the optimization functions.

1.1 Unconstrained Minimization —— 10 Marks

1. One of the functions is **fminunc** for *unconstrained optimization*. Use the description/demo, and the random starting point $x0 = randn(2,1)$, to minimize the function

   $$f(x) = e^{x_1}\left(4000x_1^2 + 2x_2^2 + 4x_1x_2 + 2x_2 + 1\right) \tag{1.1}$$

   Provide the starting point $x0$, the optimal solution x^* and the optimal value $f(x^*)$ to five decimals accuracy.

2. Use fminunc to minimize the function in (1.1) again, but first change the default tolerances using **optimset**, i.e. to see all the options enter

   optimoptions

   First, turn off the large-scale algorithms (the default). Enter

   options = optimset('LargeScale','off') ;

Next, override the default termination criteria. First, generate a random starting point $x00 = randn(2, 1)$, and compare the optimal solution, the optimal value, and the number of iterations, using the same starting point and under the following three settings.

(a)

x0=x00;options = optimset(options,'TolX',1e-2,'TolFun',1e-2);

(b)

x0=x00;options = optimset(options,'TolX',1e-3,'TolFun',1e-3);

(c)

x0=x00;options = optimset(options,'TolX',1e-8,'TolFun',1e-8);

1.2 Plots —— 5 Marks

Submit contour and surface plots of the function f for variable values between $-8, +8$. (See the commands: meshgrid, surf, plot, or demo matlab graphics, or www.math.neu.edu/~Braverman/Teaching/Fall2000/surface-contour.pdf.)

1.3 Miscellaneous Notes

Note: You can modify the MATLAB programs. You can see what the MATLAB routines are doing by looking at the MATLAB m-files. To do this change directory using: cd /software; cd matlab; cd distribution; cd toolbox; cd optim. In particular, there is an m-file called bandem.m. This provides a demo for minimization of the *banana function*.

2 Linear Algebra/Calculus Review

2.1 Quadratic Forms —— 10 Marks

1. Express the following quadratic function in the form $x^T Ax + b^T x + \alpha$ in two ways: (i) with A symmetric; and (ii) with A nonsymmetric.

$$f(x) = 3x_1^2 - 8x_2^2 + 2.1x_3^2 + 4x_1 - 4.2x_1x_2 + x_2 + 3.7x_1x_3 + 8.1x_2x_3 - 3.1x_3 + 9.4$$

2. Compute the gradient $\nabla f(x)$ and Hessian $\nabla^2 f(x)$ of the quadratic function in part 1.

3. If $f(x)$ is a quadratic function of n variables such that the corresponding matrix A is positive definite, show that $0 = 2Ax + b$ has a unique solution and that this solution is the strict global minimizer of $f(x)$.

2.2 Quadratic Forms —— 7 Marks

Classify the following matrices according to whether they are positive or negative definite or semidefinite or indefinite. (Show your calculations.)

1.
$$\begin{pmatrix} 900 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

2.
$$\begin{pmatrix} 7 & 0 & 0 \\ 0 & -19 & 0 \\ 0 & 0 & 2 \end{pmatrix}.$$

3.
$$\begin{pmatrix} -71 & 1 & 1 \\ 1 & -2 & 0 \\ 1 & 0 & -5 \end{pmatrix}.$$

4.
$$\begin{pmatrix} 18 & 11 & 15 \\ 11 & 7 & 11 \\ 15 & 11 & 16 \end{pmatrix}.$$

5.
$$\begin{pmatrix} 3 & 1 & 2 \\ 1 & 5 & 3 \\ 2 & 3 & 7 \end{pmatrix}.$$

6.
$$\begin{pmatrix} -4 & 0 & 1 \\ 0 & -3 & 3 \\ 1 & 3 & -8 \end{pmatrix}.$$

7.
$$\begin{pmatrix} 2 & -4 & 0 \\ -4 & 8 & 0 \\ 0 & 0 & -3 \end{pmatrix}.$$

3 Critical Points —— 10 Marks

(Text: Problem 7, page 32)
Use the principal minor criteria to determine (if possible) the nature of the critical points of the following functions:

1.
$$f(x_1, x_2) = x_1^3 + x_2^3 - 3x_1 - 12x_2 + 20.$$

2.

$$f(x_1, x_2, x_3) = 3x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_2x_3 + 2x_1x_3.$$

3.

$$f(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2 - 4x_1x_2.$$

4.

$$f(x_1, x_2) = x_1^4 + x_2^4 - x_1^2 - x_2^2 + 1.$$

5.

$$f(x_1, x_2) = 12x_1^3 - 36x_1x_2 - 2x_2^3 + 9x_2^2 - 72x_1 + 60x_2 + 5.$$

4 Global Maximizer —— 8 Marks

(Text: Problem 16, page 33)

1. Show that no matter what value of a is chosen, the function

$$f(x_1, x_2) = x_1^3 - 3ax_1x_2 + x_2^3$$

   has no global maximizers.

2. Determine the nature of the critical points of this function for all values of a.