

Syllabus C&O 367, W08

Nonlinear Optimization

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1 Unconstrained Optimization via Calculus

1.1 First and Second Order, Necessary and Sufficient, Optimality Conditions

(understand the different cases; understand examples for the different cases)

1.2 Local and Global Minimizers

also existence, e.g. coercivity, strict convexity; special case of quadratic functions

1.3 (Non)Attainment

(attainment and nonattainment; coercivity)

1.4 Role of positive (semi)definite matrices

in optimality conditions (and examples); Hessians

2 Convex Sets and Functions

2.1 Basic Definitions

Including cones, convex hull, polar cones

2.2 First and Second Order Characterizations of Convex Functions

Using the gradient and Hessian, respectively.

2.3 Unconstrained Geometric Programming

3 Iterative Methods for Unconstrained Optimization

3.1 First Order Methods

Method of Steepest Descent

3.2 Rates of Convergence

3.3 Newton's Method

4 Convex Programming and the Karush-Kuhn-Tucker (KKT) Conditions

4.1 Hyperplane Separation Theorems

4.2 Proof of KKT Conditions

4.3 Necessity/Sufficiency, Constraint Qualifications, Attainment

4.4 Duality

Lagrangian Dual, Wolfe Dual, Examples (LP, QP, SDP), Connection between duality and the KKT conditions.

4.5 Lagrangian Relaxation

Max-Cut example

4.6 Proof of KKT and Weakest Constraint Qualifications

cone of feasible directions, tangent cone, linearizing cone, using the weakest constraint qualification to prove the KKT conditions for a general convex program (allowing for affine equality constraints); extensions of the KKT conditions to general nonconvex programs.

5 Penalty and Barrier Methods

5.1 L1 and L2 Penalty Methods for Equality and Inequality Constraints

5.2 Barrier method for Inequality Constraints