C&O 367
Midterm Feb. 27, 2003.

Instructor H. Wolkowicz
5 questions total 65 marks; and 1 bonus question 10 marks.
Exam is from 10:00AM to 11:20AM.
NO CALCULATORS.

1 (15 marks)

Let g be a convex, monotonically nondecreasing function of a single variable (that is $g(y) \leq g(\bar{y})$, for $y < \bar{y}$), and let f be a convex function defined on a convex set $C \subset \Re^n$. Show that the function h defined by

$$h(x) = g(f(x))$$

is convex on C. Use this fact to show that the function $h(x) = e^{\beta x^T Q x}$, where β is a positive scalar and Q is a positive semidefinite symmetric matrix, is convex on $\Re^n$.

2 (10 marks)

Show that the function $f(x) = (x_2 - x_1^2)^2 + x_1^5$ has only one critical (stationary) point. Decide whether this is a local maximum or a local minimum or neither.

3 (10 marks)

Consider the function $f(x) = 5x_1^2 + 5x_2^2 - x_1x_2 - 11x_1 + 11x_2 + 11$. Perform one iteration of steepest descent with exact line search from the point $x = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. (Show your work carefully.)

4 (20 points)

1. State Newton's method for minimizing the function $f : \Re^n \rightarrow \Re$.

2. Consider the function

$$f(x_1, x_2) = x_1^4 + 2x_1^2 x_2^2 + x_2^4.$$

   (a) Evaluate the gradient and Hessian of f.

   (b) Evaluate the Newton direction and iterate from the initial point $x^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

   (c) What point does Newton's method converge to and at what rate? Can you explain the reason for this convergence rate?

5 (10 points)

Consider the quadratic function on $\Re^n$

$$q(x) = \frac{1}{2}x^T Ax - b^T x,$$

where $A = A^T$. State and prove conditions that show q is a convex function. (You may use any of the know characterizations for convexity, e.g. Jensen's inequality, gradient inequality, Hessian condition.)

6 BONUS (10 points)

1. Suppose that q is as defined in question 5 above. State and prove conditions when q is bounded below.

2. Now, suppose that q is bounded below. Prove that the minimal value is attained.