Comprehensive Exam Syllabus for
Continuous Optimization

Examiners: TBA

Suggested References:

[BV] S. Boyd and L. Vandenberghe, *Convex Optimization*, [QA402.5.B69 2004]

[NW] J. Nocedal and S. Wright, *Numerical Optimization*, 2nd Edition, 2006.

[HL] J.-B. Hiriart-Urruty and C. Lemaréchal, *Fundamentals of Convex Analysis*

Outline of Topics:

1. Linear programming, simplex method, duality (NW §§13.1–13.5 or refer to CO250 or CO255 lecture notes).
2. Optimality conditions: first- and second-derivative conditions for unconstrained optimization; KKT conditions for constrained optimization (NW §2.1, NW §§12.1–12.3)
3. Convex sets and supporting hyperplanes (BV §§2.1–2.3, 2.5; HL §A.1, §A.4.1, §A.4.2), affine hulls, relative interiors, tangent and normal cones (HL §A.2, §A.5).
4. Convex functions (BV §3.1, HL §B.1), sublinear functions and norms (HL §C.1), support functions (HL §C.2), differentiability and subgradient calculus (HL §B.4, §D.1, §D.4), Fenchel-Legendre conjugates and duality (BV §3.3; HL §E.1).
5. Canonical forms of convex optimization problems (BV §§4.1–4.4, 4.6).
6. Lagrange multipliers, Lagrangian duality, KKT optimality conditions, minmax theory (BV §§5.1–5.5).
7. Algorithms for convex optimization: Ellipsoid method and computational complexity of convex optimization (Refer to Y. Nesterov, *Introductory Lectures on Convex Programming*, Vol. I, §3.2.6, available on-line.)
8. Unconstrained optimization: basic first and second order algorithms including steepest descent and Newton's method, quasi-Newton methods (no memorization of formulae of updates is required), conjugate gradient methods; sufficient decrease criteria convergence rates; convergence theorems for Newton's method, line search and trust region methods (global convergence analysis) (NW pp. 21–23, §§3.1–3.3, §§4.1–4.2, Ch. 5, §§6.1–6.2).
9. Complexity Theory: The classes P and NP, NP-completeness. (Cook, Cunningham, Pulleyblank and Schrijver, *Combinatorial Optimization*, pages 309–323).